Analytical Model for Calculating Fault Current Contribution of a Single Phase DQ-Controlled Inverter

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***Abstract*—Based on reasonable approximations of the controller response we derive a simple yet accurate analytical model for the fault contribution of a single phase dq-controlled inverter. The derived model is compatible with typical fault calculation programs.**

***Index Terms*—dq-current control, fault, inverter.**

I. Introduction

The increasing penetration of inverter interfaced distributed energy resources, and the potential transition of such resources from anti-islanding to low voltage ride-through has created considerable interest in modeling the fault contribution of these resources in conventional short circuit analysis programs. The fault contribution is determined by the control architecture, generally limited to 110%-150% of rated current and is generally not sustained past a cycle or two. While report [1] provides useful test data, [2] proposes a Norton model with a current limit where the Norton impedance is derived from a transfer function of the inverter control scheme. Studies reported in [3] concluded that the inverter model can be represented as a constant current source equal to the pre-fault inverter current. This paper derives a simple model based on the approximate response of the current controller during a fault.

A. Inverter Model

We consider a single phase, dq-controlled inverter [4] as shown in Fig. 1 which consists of: the DC bus formed by V_{DC} and C_{DC}; the H-bridge and its filter inductance L_i; the dq-control scheme formed by an in-quadrature phase locked loop [5] and two PI controllers; the sinusoidal PWM generator; and the infinite bus with its respective transmission line impedance L_g. Inductor L_f represents a feeder. Lower case symbols such as e represent instantaneous quantities, while uppercase such as E are the corresponding phasors. Subscripts d and q represent components in the synchronous reference frame. The dynamics of the maximum power point tracker are neglected due to the fact that its time constant is much larger than the dynamics of the dq-current control scheme.

II. Derivation and analysis of short circuit model

A. Derivation

The proposed model of the inverter under fault conditions is based on two assumptions that were observed in time domain simulations of the inverter with fault locations x1, x2, and x3 in Fig. 1:

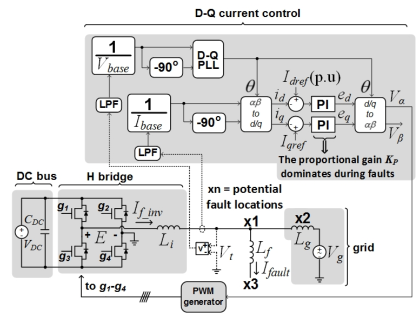

Fig. 1. Grid-tied single phase d-q current controlled inverter

1) The system frequency corresponds to the PLL center frequency (60 Hz) since during a fault occurrence there is not a significant frequency deviation in the PLL, as shown in Fig. 2-A. The frequency dip right after the fault is due to the $-90°$ delay blocks that calculate the in-quadrature components of current and voltage.
2) As seen in Fig. 2-B, the inverter fault current creates transients in e_d and e_q which settle to new but constant values. This observation suggests that in the 5-10 cycles before the fault is cleared, the proportional action of the PI controller dominates over e_d and e_q by keeping them flat constant and without any significant ramp (slope) that might be commanded from the integral action.

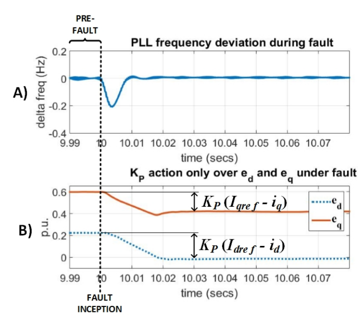

Fig. 2. Simulations results that validate the two assumptions on which the proposed inverter fault model is based

B. Analysis

The following equations model the response of the current PI controllers in Fig. 1:

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} = K_P \left(\begin{bmatrix} I_{dref} \\ I_{qref} \end{bmatrix} - \begin{bmatrix} i_d \\ i_q \end{bmatrix} \right) + \begin{bmatrix} e_{d0} \\ e_{q0} \end{bmatrix} \tag{1}$$

where e_{d0} and e_{q0} are the pre-fault values of e_d and e_q, which are the per unit dq elements of the fundamental component of the internal inverter voltage phasor E. I_{dref} and I_{qref} are the corresponding dq current set points.The inverter current in

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phasor form is: $I_{f\_inv} = \frac{E-V_t}{j\omega L_i}$, this equation can be written in dq form as:

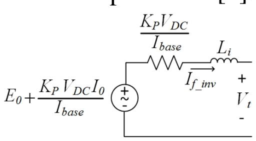
$$\begin{bmatrix} i_d \\ -i_q \end{bmatrix} = \frac{V_{DC}}{\omega \cdot L_i \cdot I_{base}} \begin{bmatrix} e_q \\ e_d \end{bmatrix} - \frac{V_{base}}{\omega \cdot L_i \cdot I_{base}} \begin{bmatrix} v_{tq} \\ v_{td} \end{bmatrix} \quad (2)$$

where v_{td} and v_{tq} are the dq components of the voltage at the inverter's terminals; V_{base} and I_{base} are the AC base quantities of the system; and V_{DC} is the DC bus voltage. Note that the aforementioned dq components are internal to the inverter controls and must be scaled accordingly.

By solving (1) and (2) an analytical expression can be found for i_d and i_q, giving:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} K_P & \frac{-\omega L_i I_{base}}{V_{DC}} \\ \frac{\omega L_i I_{base}}{V_{DC}} & K_P \end{bmatrix}^{-1} \begin{bmatrix} e_{d0} + K_P I_{dref} - \frac{V_{base} v_{td}}{V_{DC}} \\ e_{q0} + K_P I_{qref} - \frac{V_{base} v_{tq}}{V_{DC}} \end{bmatrix} \quad (3)$$

therefore, the fault current contribution from the inverter can be calculated using $I_{f\_inv} = (i_d + j i_q) I_{base}$ which, after some algebraic simplification gives (4) along with its circuit representation shown in Fig. 3, where: $E_0 = V_{DC}(e_{d0} + j e_{q0})$, $I_0 = I_{base}(I_{dref} + j I_{qref})$, and $V_t = V_{base}(v_{td} + j v_{tq})$, with v_{td} and v_{tq} values provided by the PLL. It is important to point out that if $\left(\frac{K_P V_{DC}}{I_{base}}\right) >> \omega L_i$, Equation 3 reduces to $I_{f\_inv} = I_0$ (i.e. pre-fault current), as reported in [3]. Conversely if $\left(\frac{K_P V_{DC}}{I_{base}}\right) << \omega L_i$, then $I_{f\_inv} = \frac{E_0 - V_t}{j\omega L_i}$, therefore the inverter now acts like a conventional source, but the current is limited to some preset value as reported in [2].



$$I_{f\_inv} = \frac{E_0 + \left(\frac{K_P V_{DC} I_0}{I_{base}}\right) - V_t}{\left(\frac{K_P V_{DC}}{I_{base}}\right) + j\omega L_i} \quad (4)$$

Fig. 3. Inverter model for short circuit studies

III. Cases of Study

This section compares the inverter fault current contribution I_{f_inv} calculated from the proposed model to results from time-domain simulation. Table I summarizes the key parameters of the simulations performed in SimPowerSystems™. Faults at locations x1, x2 and x3 are considered for initial operating conditions of rated power at unity power factor and roughly half-power at 0.9 power factor lagging, respectively. For each case the values of the d and q current components and the ac phasor fault contribution are compared as shown in Tables II to IV. It is seen that in all cases the model results closely match the results from simulation.

TABLE I
Simulation parameters for single phase DQ inverter

Parameter	Value	Parameter	Value
V_{DC}=V_{baseDC}	400 volts	L_g	1.5 mH
V_g=V_{baseAC}	240 volts	PI for d and q	$K_P = 5$ $K_I = 7$
I_{base}	170 amps	PI for PLL	$K_P = 4$ $K_I = 7$
L_i	2.1 mH	VCO center frequency	60 Hz
L_f	1.5mH	C_{DC}	$10000\mu F$

IV. Conclusion

A simplified analytical model of a single phase, dq-controlled inverter, that bridges the models in [2] and [3] is proposed. For inverters with ride-through control this model is applicable to the 2-5 cycles prior to this control becoming effective. The model clearly shows why the fault contribution is often comparable to the pre-fault current. Also, the model clearly displays the relation between fault contribution and inverter parameters along with pre-fault conditions. Furthermore, the model can be easily incorporated in a short circuit analysis program. The model does require knowledge of the parameters L_i and K_P. We acknowledge that detailed data on inverters is generally not available, but we suggest that manufacturers could provide a value for the effective short-circuit impedance as in Fig. 3.

TABLE II
Results comparison for a fault at inverter's terminals

Simulation	Analytical model
case: $\mathbf{I_{dref} = 0.9}$ p.u and $\mathbf{I_{qref} = 0}$ p.u	
i_{d_sim} =0.983 p.u	i_{d_model} =0.95 p.u
i_{q_sim} =0.065 p.u	i_{q_model} =0.06 p.u
$I_{f\_inv\_sim} = 0.98\angle 3.7°$ p.u	$I_{f\_inv\_model} = 0.95\angle 3.6°$ p.u
case: $\mathbf{I_{dref} = 0.5}$ p.u and $\mathbf{I_{qref} = 0.2}$ p.u	
i_{d_sim} =0.579 p.u	i_{d_model} =0.57 p.u
i_{q_sim} =0.263 p.u	i_{q_model} =0.25 p.u
$I_{f\_inv\_sim} = 0.63\angle 24.43°$ p.u	$I_{f\_inv\_model} = 0.62\angle 23.7°$ p.u

TABLE III
Fault on transmission line at a distance 25% of line length from inverter

Simulation	Analytical model
case: $\mathbf{I_{dref} = 0.9}$ p.u and $\mathbf{I_{qref} = 0}$ p.u	
i_{d_sim} =0.99 p.u	i_{d_model} =0.96 p.u
i_{q_sim} =0.057 p.u	i_{q_model} =0.05 p.u
$I_{f\_inv\_sim} = 0.99\angle 3.2°$ p.u	$I_{f\_inv\_model} = 0.96\angle 3°$ p.u
case: $\mathbf{I_{dref} = 0.5}$ p.u and $\mathbf{I_{qref} = 0.2}$ p.u	
i_{d_sim} =0.586 p.u	i_{d_model} =0.57 p.u
i_{q_sim} =0.259 p.u	i_{q_model} =0.243 p.u
$I_{f\_inv\_sim} = 0.637\angle 23.8°$ p.u	$I_{f\_inv\_model} = 0.62\angle 23.1°$ p.u

TABLE IV
Results comparison for a remote fault

Simulation	Analytical model
case: $\mathbf{I_{dref} = 0.9}$ p.u and $\mathbf{I_{qref} = 0}$ p.u	
i_{d_sim} =0.943 p.u	i_{d_model} =0.904 p.u
i_{q_sim} =0.032 p.u	i_{q_model} =0.02 p.u
$I_{f\_inv\_sim} = 0.94\angle 1.9°$ p.u	$I_{f\_inv\_model} = 0.904\angle 1.3°$ p.u
case: $\mathbf{I_{dref} = 0.5}$ p.u and $\mathbf{I_{qref} = 0.2}$ p.u	
i_{d_sim} =0.541 p.u	i_{d_model} =0.523 p.u
i_{q_sim} =0.231 p.u	i_{q_model} =0.22 p.u
$I_{f\_inv\_sim} = 0.588\angle 23.12°$ p.u	$I_{f\_inv\_model} = 0.57\angle 22.8°$ p.u

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