On Extended-Term Dynamic Simulations with High Penetrations of Photovoltaic Generation

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***Abstract*—The uncontrolled intermittent availability of renewable energy sources makes integration of such devices into today's grid a challenge. Thus, it is imperative that dynamic simulation tools used to analyze power system performance are able to support systems with high amounts of photovoltaic (PV) generation. Additionally, simulation durations expanding beyond minutes into hours must be supported. This paper aims to identify the path forward for dynamic simulation tools to accommodate these needs by characterizing the properties of power systems (with high PV penetration), analyzing how these properties affect dynamic simulation software, and offering solutions for potential problems. In particular, the system eigenvalue configuration of representative power system models is examined and how this configuration influences numerical integration scheme selection is discussed.**

I. Introduction

In the quest for a clean and sustainable future, there exists a large push towards incorporating substantial amounts of renewable energy sources such as photovoltaic (PV) generation. The uncontrolled intermittent availability of renewable energy sources makes integration of such devices into today's grid very challenging. Technical issues include energy and power balancing, voltage regulation and stability, frequency regulation, transient stability, and small-signal stability. Another challenge is that the characteristics of a grid with high PV penetration, e.g. 100% of load, will have dynamics significantly different from the grid of today. Currently, transient simulations capture the electro-mechanical response of the grid to various disturbances. A grid dominated by inertia-less generation (e.g. renewables with inverters) will potentially be more responsive to disturbances. The goal of this study was to develop a path forward for dynamic simulation tools that enable analysis of power system performance (with high PV penetration) for a period of minutes to hours. Our focus was to examine the fundamental drivers, the algebraic and differential equations that model a grid with 100% PV generation, to identify the path forward for dynamic simulation tools that support high renewables as well as longer simulation times.

The topic of extended-term time-domain simulation for electric power systems is beginning to garner increasing attention in the literature. In [1], the authors proposed an integration method called Hammer-Hollingsworth 4 (HH-4), which is a special case of the implicit fourth order Runge-Kutta method that is A-stable, possesses the same stability domain as the Trapezoid Rule (2nd-order Adams-Moulton method), and has a higher order of accuracy than the Trapezoid Rule [2].

A numerical method is said to be A-stable if all of its solutions to equations of the form

$$\frac{dy}{dt} = ky, k \in \mathbb{C} \tag{1}$$

where

$$y(t) = Ae^{kt} \; \forall \; \mathrm{Re}(k) < 0 \tag{2}$$

decay to zero as $t \to \infty$ [3]. This means that for differential equations for which the true solution decays to zero as a function of time, the numerical solution also decays, rather than diverging. Equivalently, a method is A-stable if its region of stability contains all of the left half-plane [3]:

$$\text{Region of Stability} \supseteq \{h\lambda \in \mathbb{C} \mid \mathrm{Re}(h\lambda) < 0\} \tag{3}$$

where h represents the simulation step size and λ represents the continuous-time system eigenvalues.

Because the HH-4 method is implicit, the state update equations constitute a nonlinear system which must be solved iteratively. This makes the method much more computationally intensive than linear multistep methods and predictor-corrector schemes and dependent on the specific set of differential equations. Additionally, all fourth order Runge-Kutta methods. including HH-4, require the calculation of the state derivatives to be performed four times per integration step. In contrast, a predictor-corrector scheme based on the Trapezoid Rule requires the state derivatives to be calculated only twice. The region of stability of the Trapezoid Rule is ideal because it includes all of the left half of the complex plane, and none of the right. However, the Trapezoid Rule is also an implicit method, which makes it nontrivial to implement in software in addition to its computational challenges. The integration techniques collectively called predictor-corrector methods serve as a compromise in which the solution to an implicit method is approximated using purely explicit formulations [4].

At present, the standard commercial tools for performing time-domain simulation of large-scale power systems employ explicit, multistep numerical integration methods with a fixed step size. The integrator employed by PSLF and PSS/E, the second order Adams-Bashforth method (AB-2), has a region of stability that is a subset of the left half of the complex plane. This means that the currently employed numerical integration schemes have the potential to exhibit numerical instability for stable systems [5].

An ideal numerical integration scheme for dynamic simulation purposes would possess a larger region of stability and a higher order of accuracy than AB-2. An intelligently chosen predictor-corrector scheme could satisfy both criteria. Since predictor-corrector schemes are explicit formulations, they cannot be A-stable like the Trapezoidal Rule [6]. However, they can possess a significantly larger region of stability than AB-2, allowing for larger simulation step sizes [7].

The practical implication of this is that the choice of step size for an explicit integration scheme will impact whether or not it exhibits numerical instability. However, numerical stability cannot be the only consideration for integrator selection. There is an inherent trade-off between numerical accuracy and computational workload when the step size of a simulation is modified; in general, simulations run faster at the expense of accuracy with larger step sizes. For explicit methods, the step size must be tuned appropriately such that the eigenvalues of the system reside within the region of stability. Therefore, it is essential to understand the eigenvalue topology of typical power system models, possibly with very high PV penetration, in order to make the best compromise on numerical integrator selection that makes extended-term simulations viable.

The rest of the paper is organized as follows. Section II describes how we modeled PV generation and how we determined the properties of power systems with high penetrations of PV drive the selection of an numerical integrator. Section III describes the candidate integrators that we have identified and the inherent tradeoff among them. Concluding remarks follow in section IV.

II. Characteristics of Systems with High PV Penetration

We employed Power Systems Toolbox (PST) for MATLAB [8] as a test and development platform for this effort due to the ability to modify the code. We implemented different explicit integrator schemes as well as a custom model for PV generation based on a current injection model.

A. Modeling PV Generation

To model increasing PV penetration, PV generation models are co-located with existing traditional generation in a PST test case. Each bus representing PV generation shares a point of connection to the rest of the system with its co-located traditional generation. A solar fraction parameter is used to shift a fraction of generated power from the original generator to the photovoltaic generator. The total amount of active power generated is conserved between the two sources such that the aggregate active power injected into the common point of connection is constant across all solar fraction values. However, only 50% of the reactive power shifted from the original generator is supplied by the photovoltaic source. For example, if 20% of power is shifted to solar generation, only 10% of the reactive power is shifted to the reactive power specified for the photovoltaic generation bus. Machine inertia specified in the dynamic record is reduced by proportionally scaling down the mVA base of the synchronous machines.

B. Stiffness Analysis

Initially, it was hypothesized that an increase in PV penetration would increase system stiffness. One way to measure system stiffness is using the stiffness ratio, defined as:

$$\text{stiffness ratio} = \frac{\max|\text{Re}(\lambda)|}{\min|\text{Re}(\lambda)|} \tag{4}$$

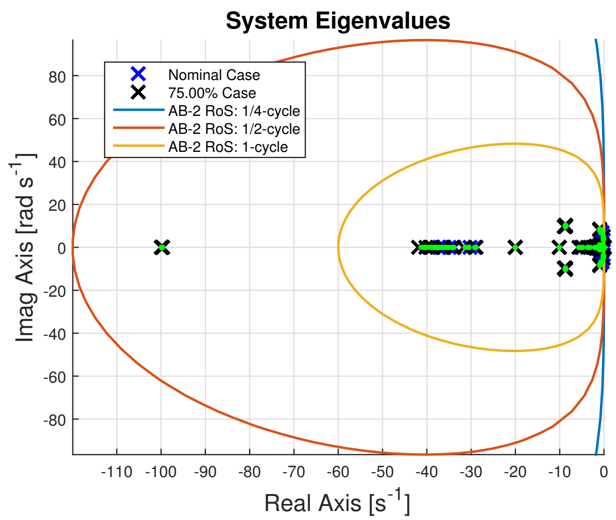

Fig. 1: System eigenvalues for a four machine, 16 bus test case with the region of absolute stability for AB-2 for various step sizes superimposed.

This property of a system of differential equations roughly describes the range of dynamics present in the system; a large stiffness ratio implies that there are modes with very fast decay rates, very slow decay rates, or a combination of both. Intuitively, this represents a type of difficulty in integrating the associated differential equations; both fast and slow dynamics need to be accounted for. More explicitly, the system eigenvalues of a dynamical system can be compared to the region of absolute stability for a given integrator. In Fig. 1, the system eigenvalues for a four machine, 16 bus test case are plotted across various solar fraction values. These system eigenvalues were estimated using small signal stability analysis tools in PST. The regions of absolute stability for AB-2 for various step sizes are superimposed; as the step size increases, the region of absolute stability compresses and the integrator becomes less accommodating for faster dynamics as expected.

However, analysis of various test cases showed there to be no correlation between PV penetration and system stiffness ratio. Fig. 2 zooms in on the slower, low frequency modes from Fig. 1. Although many system eigenvalues tend to drift left in the s-plane as PV penetration increases, it is unlikely that this will increase system stiffness because these eigenvalues rarely correspond to the fastest dynamics in the system. Therefore, it is unlikely that increased PV penetration (as we have modeled

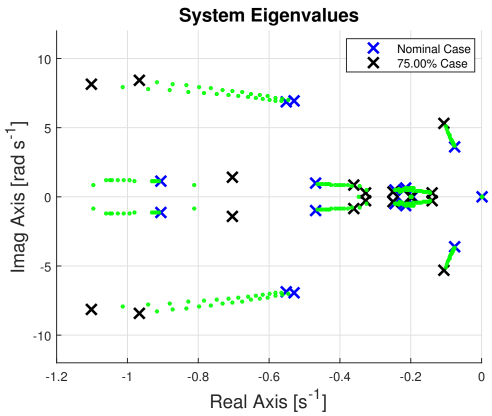

Fig. 2: System eigenvalues for a 16 bus test case with region of absolute stability for AB-2 for various step sizes superimposed.

it), would play a role in numerical integrator selection on its own.

By utilizing the linearization capabilities of PST, we were able to identify how different models and their dynamics in a power system test case stress integrator stability. The mode shapes and participation factors associated with each estimated system eigenvalue describe which model components are responsible for each eigenvalue. For example, from Fig. 1, the fast, non-oscillatory modes located on the negative real axis would be the limiting factor in integration selection due to numerical stability; these eigenvalues were found to be directly related to excitation system time constants which were in this case set to 10 ms. Furthermore, these eigenvalues were observed to remain fixed regardless of PV penetration. Consequently, since they are typically the modes with the fastest decay rates in a given system, this is why system stiffness does not tend to increase with PV penetration.

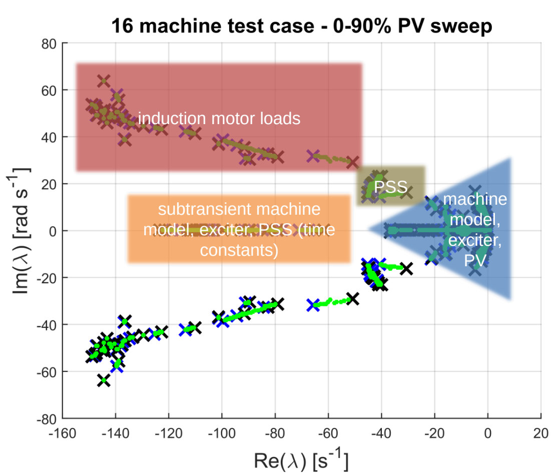

Fig. 3: System eigenvalue topology of an archetypical power system model. (16 machine case)

Fig. 3 shows a system eigenvalue map with a sweep across a range of solar fraction values up to 90% for a larger, 16 machine test case. Overlays describing what system components are primarily responsible for each region of eigenvalues based on our aforementioned analysis are also included. Notably, induction motor loads tend to produce higher frequency, faster modes; we observed that these modes tend to drift left with increased PV penetration. Therefore, the inclusion of these loads in a system model would play a significant role in integrator and step size selection. This figure is discussed in more depth in the following section.

III. Candidate Integrator Analysis

In our initial investigations with PST, we learned that the toolbox relies upon a 2nd-order accurate predictor-corrector algorithm known as Heun's method [9]. This integration scheme uses the forward Euler scheme as its predictor and the Trapezoid Rule as its corrector. To achieve the goals of this study, we investigated the behavior of the integration scheme employed by both PSLF and PSS/E, AB-2. Based on the analysis of system stiffness and other computational requirements, we identified the 4th-order accurate Crane-Klopfenstein (CK-4) predictor-corrector scheme as a candidate explicit integration scheme [10]. The CK-4 integration scheme possesses a high order of accuracy and excellent stability characteristics while being straightforward to implement in software. As a baseline, we included the simple Forward Euler integration scheme although it is not a real candidate due to its limited stability properties and poor accuracy.

A. *Computational Considerations*

When looking at the computational burden of integration schemes, we primarily look at the number of "rate" calls and number of memory storages and calls per time step or iteration. "Rate" calls are the execution of the routine to compute the derivatives of the state variables in the system. Typically, this is only once per iteration for standard explicit integration schemes but predictor-corrector schemes can include numerous rate calls. Memory access is mostly tied to the order of the integration scheme but can also increase depending on the implementation of a predictor-corrector scheme.

Table I: Summarizing the number of memory and rate calls for candidate integration techniques.

	Memory calls	Rate calls
Forward Euler	1	1
AB-2	2	1
Heun's method	3	2
CK-4	13	2

B. *Integrator performance benchmark tests*

In order to demonstrate how computational differences among the integrators affect real time performance, we developed a benchmarking tool in MATLAB. Using a simple, linear second order differential equation test system with a single complex eigenvalue pair, we simulated a step response using each of the integrators of interest. We performed a 100 second simulation for 3 different step sizes; this means that the number of steps in each simulation varied depending on the step size. We simulated 100 different systems in which the eigenvalue pair location each time was randomized but within the region of absolute stability for all integrators. The simulations were performed on a computer with an Intel Core i7-4600U CPU @ 2.1 GHz and 8.00 GB of RAM running Windows 7. The results, in seconds, are shown in Table II.

Table II: Total time taken to complete 100 simulations for various step sizes.

	Total Time [s]		
	$h = \frac{1}{4}$ **cycle**	$h = \frac{1}{2}$ **cycle**	$h = 1$ **cycle**
Forward Euler	19.50729	9.320980	4.744207
AB-2	22.70859	10.77197	5.325676
Heun's method	33.04143	15.54170	7.731683
CK-4	50.57874	23.36001	11.599648

In order to see how average simulation time scaled with step size/step count, we performed a similar experiment with a broader range of step sizes. The results are shown in the Fig. 4. We see that average simulation time scales roughly exponentially with step count. These results are interesting in the context of the other factors driving integrator selection. For example, the commonly used AB-2 has fairly good computational performance for the standard quarter cycle step size (0.004 s). As discussed in the following section, the candidate CK-4 has similar, if not more desirable, numerical stability properties as AB-2 at the full cycle step size (0.016 s). If we extrapolate from the previous benchmark, CK-4 is faster than AB-2 when considering the different step size. If the accuracy is acceptable for CK-4 at this larger step size and the integrator is numerically stable, it would favor selecting CK-4 for simulation.

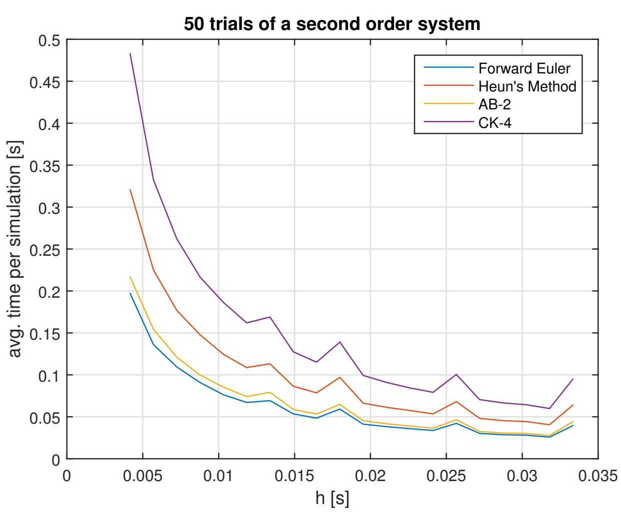

Fig. 4: Average time per 100 s simulation taken over 50 trials.

C. Numerical Stability Considerations

With numerical stability as the priority criterion for selecting an integrator due to its "pass/fail" nature, it is vital to understand the eigenvalue topology for the typical power system to be simulated as we analyzed in Section II. As we observed, power system models typically contain the same component dynamic models with associated system eigenvalues in the same region of the complex plane; there is variation in eigenvalue location due to actual parameter values. In order to understand how numerical integrator selection relates to power system eigenvalue topology, we analyzed where each integrator's region of absolute stability lies in relation to the eigenvalue topology map from Section II.

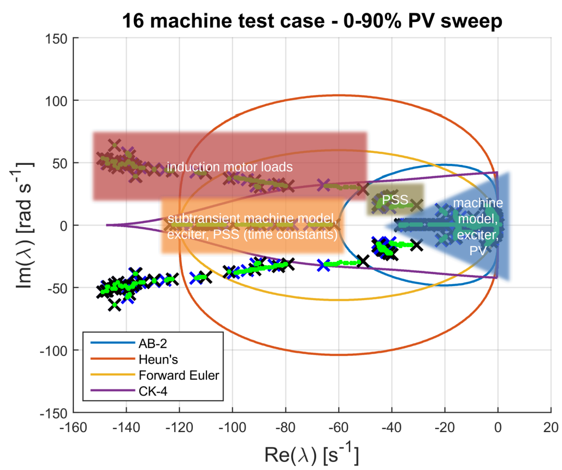

Fig. 5: System eigenvalue topology with region of absolute stabilities for $h = \frac{1}{60}$ s overlaid.

Figure 5 shows the resulting illustration for a full cycle step size. Using the aforementioned methodology for mode identification and state association, we annotated the eigenvalue map to indicate what dynamic models are associated with various regions in the complex eigenvalue plane. As noted previously, induction motor load models are most likely to restrict the selection of an integrator due to their fast decaying, high frequency modes. Most commonly, the time constants associated with transient and subtransient machine models, exciters, and PSS will stress the selection of an integrator and/or step size. The region annotated on the eigenvalue topology map is directly correlated to these time constants, which are typically in the 20 ms or smaller range. The dynamics associated with these time constants are far and away the fastest dynamics in power system models that do not contain induction motor loads. Due to region of absolute stability shapes for typical explicit integration schemes, these time constants will most likely restrict how large the step size can be. The other two regions identified are highly unlikely to affect the choice of integrator and step size; these relatively slower decaying, low frequency modes will almost surely be well within the region of absolute stability for any integrator unless all of the aforementioned time constants happen to be very large. As noted in Section II, increased PV penetration, as modeled, has no definite effect on system stiffness. Integrator selection stress does not directly come from the presence of PV-related current injections, but rather from the tendency for system eigenvalues to drift left with increased PV penetration.

D. Integrator Analysis Conclusions

One of the reasons for considering different integrators for extended-term simulation of power systems with high PV penetration is that for very long simulation lengths, it is less feasible to use integrators with the oft-used quarter-cycle step size due to computation speed and data storage limitations. Based on our analysis, the AB-2 scheme with a quarter-cycle step size is very capable for simulating most power systems and is perfectly suitable for shorter duration simulations. For simulations of durations in the extended-term regime, increasing the step size to, e.g., a full cycle would be a massive improvement in terms of computation time and data storage management. Because of its unique numerical stability properties, we recommend CK-4 as an integrator because it tends to be highly compatible with many power system models; additionally, it gives more of the s-plane where it matters in terms of absolute stability for a given step size. As a result, one is most likely able to reduce the simulation step size using CK-4 compared to the other candidate integrators. While this comes at the cost of additional computation time, based on our analysis, it may actually be faster to use CK-4 than other integrators because other integrators are more likely to require a smaller step size for numerical stability reasons.

One drawback of using CK-4 is its thinner region of absolute stability in terms of frequency. From the example of the system with induction motor loads, this property tends to be problematic for CK-4 because of the existence of large decay rate, high frequency modes. The presence of these modes requires CK-4 to use a larger step size to be numerically stable when simulating this type of system and eliminates the advantage of CK-4. For these cases, we recommend using Heun's method as it contains much more frequency bandwidth for a given step size.

IV. Conclusion

In this study, we focused on improving the feasibility of extended-term dynamic simulations of power systems with very high PV penetration primarily from the perspective of numerical integration. We saw that moving into the extended-term regime presented issues such as increased computational burden and data storage use and proposed modifying how simulation software performs numerical integration in order to address these concerns. Since some of the most commonly used power system simulation software make use of the explicit second order Adams-Bashforth integration method, we investigated other explicit integration methods due to their relative ease of implementation.

Since numerical stability is a primary concern for numerical integration, we analyzed the dynamic stability properties of power systems with increased PV penetration. We identified how different power system dynamic models affect system modes and what role they play in selecting an integrator. Based on our investigations, we found that while increased PV penetration does have an effect on system dynamic behavior, it is rarely a primary factor in stressing the selection of an integrator. We found that the presence of certain components, such as induction motor loads, are most often the driving force in integrator and step size selection.

We found that the fourth order Crane-Klopfenstein predictor-corrector scheme to be a viable numerical integrator because its region of absolute stability shape encompasses the entirety of typical power system eigenvalues even at increased step sizes. This potential increase in step size can produce a lot of computational and storage savings for extended-term simulations. On the other hand, in terms of numerical stability, we found that this scheme is incompatible with high frequency, fast decaying modes associated with induction motor loads. In such cases, we found that Heun's method is similarly accommodating for system eigenvalues at a given step size and is a safe alternative when the system's dynamic characteristics are unknown or problematic for CK-4.

Acknowledgment

This research was supported by the U.S. Department of Energy, SunShot Initiative, under Agreement No. 29095. Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

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