Comparison of Methods for Estimating Short-Term Extreme Response of Wave Energy Converters

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***Abstract*—Short-term extreme response statistics are often required to obtain the long-term (deployment life) response for an offshore structure. A number of methods are available to produce these response statistics for data collected from either physical experimentation or numerical modeling. Here, we consider the application of a series of such methods to determine the short-term extreme response statistics for a simple wave energy converter (WEC). Using data created from a frequency-domain model, each method is implemented multiple times to provide an understanding of its accuracy and variance. The results are compared along with an empirical "truth" result that uses all of the available data. Trade-offs between the amount of data required by a given method and its accuracy are presented and discussed.**

***Keywords*—Wave Energy, WEC, Extreme Value, Short-Term Statistics, Design Load.**

I. Introduction

A better understanding of extreme loads on wave energy converters (WEC) has the potential to increase the economic viability of WECs. More accurate design loads would allow for device designs that can survive in the harsh ocean conditions without the need for excess (wasted) structural material. This requires (i) a statistical understanding of the environmental conditions at the deployment site, as well as (ii) an understanding of the WECs response at the different environmental conditions. In general, the methods employed to analyze analogous systems (e.g., offshore oil and gas structures, ships, and offshore wind turbines) should be applicable to WECs [1], [2]. However, some care must be taken to insure that the unique aspects of a WEC do not necessitate a somewhat altered approach.

In general, the design response for an offshore system is described by a long-term (e.g. deployment life) extreme response probability distribution. This distribution is the combination of short-term (i.e specific environmental condition) extreme response probability distributions and distributions describing the environmental conditions. A short-term extreme response distribution describes "If a device is in sea-state X for Y amount of time, what will be the largest Z observed," where X is the environmental condition, Y the short-term period, and Z the response parameter of interest (e.g. the mooring load, bending moment). To move from a distribution to some scalar value that can more easily be used in the design process, the characteristic extreme value is often taken as the expected value (the mean) or some higher r^{th} percentile of the distribution. Combining a device's short-term extreme response distribution for a given sea state with the probability of occurrence for that sea state produces a long-term extreme response that can be used to quantify the design response of a device (see, e.g., [3], [4]).

An ongoing project is currently investigating the full scope of this process, however, this paper specifically looks at the methods of estimating characteristic extreme response values from numerical simulations. The goal is to compare different methods and make an informed selection to be used in the larger project. Since the process of estimating short-term extreme responses may need to be repeated possibly hundreds of times to assess the device response across a range of sea states, it is desirable for (i) the numerical model to be fast running, and (ii) for the method of estimating the extreme response distribution to require the least amount of simulation time. For this study, we consider a simple numerical model and focus our analysis on part (ii) by considering a number of statistical methods and in each case varying the amount of data used to implement the method. The results provide a relative comparison of the accuracy and efficiency trade-off between the methods. Additionally, an empirical "truth" is calculated and used to provide some absolute comparison for the methods.

II. Methods for Obtaining the Extreme Event Probability Distribution

In this study, we compare different methods for obtaining the extreme response at a given sea-state. A wide range of methods are available and are regularly applied to marine systems. Here, we consider four of the most popular methods: all-peaks Weibull, Weibull tail-fit, block maxima, and peaks-over-threshold. Each of these methods make different compromises between the amount of data used and the relevance of that data to extreme events. The tail of the distribution (i.e. the largest global peaks) are the most important when extrapolating to the short-term extreme distribution. In all cases, the global peaks (largest point between successive zero up-crossings) are identified and treated as an independent random variable.

A. All-Peaks Weibull

The all-peaks Weibull method uses a single simulation time-series of any length, not necessarily the desired short-term period (i.e. the length of the simulation may be shorter than, equal to, or longer than the desired short-term period) [5]. A Weibull distribution is assumed and fitted to the global peaks. The cumulative distribution function (CDF), $F(x)$, for a Weibull distribution is given by

$$F(x) = 1 - \exp\left(-\left(\frac{x}{\alpha}\right)^{\lambda}\right) \qquad (x \geq u). \tag{1}$$

Here, α and λ are the distribution's scale and shape parameters respectively. The short-term extreme distribution function, $F_{\mathrm{e}}(x)$, can then be defined in relation to a distribution of the response's peaks, $F_{\mathrm{p}}(x)$

$$F_{\mathrm{e}}(x) = F_{\mathrm{p}}(x)^q, \tag{2}$$

where q is the number of expected peaks in the short-term period. This method uses the most data (all global peaks), however much of this data may not be relevant to extreme events. Thus, by fitting a Weibull to all the global peaks, one risks not getting the best fit for the tail of the peak distribution, which has the largest effect on the extreme distribution.

B. Weibull Tail-Fit

The Weibull tail-fit method is similar to the all-peaks Weibull, but emphasizes the tail of the distribution [6], [7]. With the peaks in ascending order, an empirical distribution is approximated as

$$F'(x_i) = \frac{i}{N+1} \tag{3}$$

for the i^{th} ordered peak, where N is the total number of peaks. Distributions are fitted to the points $(x_i, F'(x_i))$ for seven different datasets where $F'(x_i) \geq 0.95,\ 0.90,\ 0.85,\ 0.80,\ 0.75,\ 0.70,$ and 0.65. Thus, the 1[st] distribution is a fit to the peaks with $F'(x_i \geq 0.95)$, the 2[nd] is fitted to peaks with $F'(x_i \geq 0.90)$ and so on. The distribution of global peaks is then taken as a Weibull, with parameters equal to the average of the parameters of the seven fitted Weibull distributions. The short-term extreme distribution is then obtained using (2).

By obtaining a better fit on the tail of the peaks, this method should provide more accurate estimates for the short-term extreme distribution. Mathematically, we can consider how the CDF of the peaks (which is always between 0 and 1) is raised to a large positive number, q, to obtain $F_e(x)$. This operation causes the short-term extreme CDF to shift to the right of the peak CDF, due the fact that $F(x)^{q+1} > F(x)^q$ where $q \gg 1$. Hence the section of the extreme CDF with non-negligible values comes only from the tail of the peaks CDF.

C. Block Maxima

The block maxima method is the most rigorous and straight forward method of obtaining the short-term extreme distribution. It obtains the short-term extreme distribution directly, without calculating a distribution for the global peaks. The first step is to run N simulations of length equal to the short-term period of interest. Then the extreme value (largest value observed, or block maxima) is identified from each of the N simulations. A generalized extreme value (GEV) distribution is fitted to the N extreme values. The CDF of the GEV is defined as

$$F(x) = \exp\left(-\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right) \tag{4}$$

where ξ, σ, and μ are the shape, scale, and location parameters, respectively.

Unlike the use of the Weibull distribution for fitting the global peaks, the choice of GEV is mathematically true for block maxima, regardless of the base distribution of the data [8]. This method uses only one data point per simulation, but it is the most relevant data point, however, it also requires the most simulation time.

D. Peaks-Over-Threshold

The peaks-over-threshold method uses all global peaks over a certain threshold (u). A Pareto distribution is fitted to this subset of the data. The CDF for the Generalized Pareto distribution is given by

$$F(x-u) = 1 - \left(1 + \frac{\xi(x-u)}{\sigma}\right)^{-\frac{1}{\xi}}, \tag{5}$$

where ξ and σ are the shape and scale parameters, respectively. The distribution of peaks, $F(x)_{\mathrm{p}}$, is then obtained from the distribution of peaks-over-threshold, $F(x)_{\mathrm{pot}}$, as

$$F(x)_{\mathrm{p}} = 1 - (\zeta_u(1 - F(x)_{\mathrm{pot}})), \tag{6}$$

where ζ_u is the probability of global peak being over the threshold (u) and is approximated as the fraction of observed peaks over the threshold. The short-term extreme distribution is then obtained using (2).

The Generalized Pareto distribution is mathematically true for the global peaks-over-threshold [8]. Like the Weibull tail-fit, the peaks-over-threshold method uses an intermediate amount of data. The choice of threshold is important; too low of a threshold may violate the underlying assumptions and too high decreases the amount of data available (see, e.g., [8]). Based on standard practice for offshore wind turbines, here we employ a threshold 1.4 standard deviations above the mean of the peaks [9].

III. Case-Study

For this study, the simple WEC illustrated in Fig. 1 was considered. This device is a neutrally buoyant cylinder restricted to move in heave and connected to the ocean floor via a linear damper. Table I summarizes the device's parameters. The response parameter of interest was chosen as the force on the power conversion chain (PCC), and the short-term period as one hour. The sea-state modeled consists of irregular waves represented by a JONSWAP spectrum. Spectral parameters are listed in Table I.

TABLE I. CASE-STUDY & WEC SPECIFICATIONS.

WEC Radius, R	5 m
WEC Draft, T	5 m
WEC Mass, m	402517 kg
PCC Damping Coefficient, b_{pcc}	52163 Ns/m
Water Density, ρ	1025 kg/m^3
Hydrostatic stiffness, k	786144 N/m
Ocean wave spectral family	JONSWAP
Significant wave height, H_s	6 m
Energy period, T_e	8 s
Peak period, T_p	8.85 s
Peakedness factor, γ	3.3
Response parameter of interest	PTO-Force
Short-term period of interest	1 hour

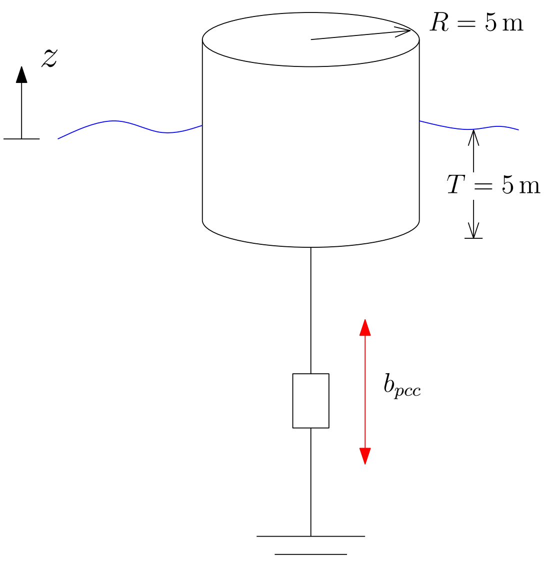

Fig. 1. Schematic of case-study WEC device.

A. Hydrodynamic Model

The WEC was modeled using a frequency-domain approach (see, e.g., [10]). The frequency-dependent excitation, $\hat{H}(\omega)$, added mass, $m_a(\omega)$, and radiation damping, $b_{rd}(\omega)$, coefficients in the heave direction were obtained using the boundary element method code NEMOH [11]. The hydrostatic stiffness is represented by a linear spring coefficient, k. The frequency-dependent wave elevation is calculated given as

$$\hat{\zeta}(\omega) = A(\omega)cos(\phi(\omega)) + A(\omega)sin(\phi(\omega))i \tag{7}$$

where $A(\omega)$ is the frequency-dependent amplitude corresponding to the JONSWAP spectrum described in Table I and $\phi(\omega)$ is the frequency-dependent random phase. The complex frequency-dependent wave excitation force is given by (8), and the WEC's impedance (intrinsic + PCC impedance) by (9).

$$\hat{F}_e(\omega) = \hat{H}(\omega)\hat{\zeta}(\omega) \tag{8}$$

$$\hat{Z}(\omega) = \omega(m + m_a(\omega))i + b_{pcc} + b_{rd}(\omega) - \frac{k}{\omega i} \tag{9}$$

The frequency-domain complex velocity is then given by

$$\hat{\dot{z}}(\omega) = \frac{\hat{F}_e(\omega)}{\hat{Z}(\omega)} \tag{10}$$

and the force in the power conversion chain (PCC), F_{pcc}, is

$$\hat{F}_{pcc}(\omega) = b_{pcc}\hat{\dot{z}}(\omega). \tag{11}$$

The time-history of the response parameter of interest is then obtained as

$$F_{pcc}(t) = b_{pcc}\sum_{i=1}^{\infty}|\hat{\dot{z}}(\omega_i)|cos(\omega_i t + arg(\hat{\dot{z}}(\omega_i))). \tag{12}$$

The damping coefficient, b_{pcc}, for the WEC was chosen by optimizing the absorbed power, P_a.

$$\begin{gathered} b_{pcc} = \text{argmax}\left\{\sum_{i=1}^{\infty} P_a(\omega_i)\right\} \\ P_a(\omega) = \frac{1}{2}b_{pcc}|\hat{\dot{z}}(\omega)|^2 \end{gathered} \tag{13}$$

B. Comparison of Statistical Method Performance

The frequency-domain model described in Section III-A was used to simulate 1000 hours of the response parameter of interest (F_{pcc}) at a specific sea-state. To reduce the number of frequencies required to generate such a long time history without repetition, the 1000 hours were obtained by concatenating the results from multiple simulations of shorter repeat period. This was possible because of the ergodic nature of the system. The global peaks between zero up-crossings were then identified and treated as an independent random variable.

For this study, the desired short-term period is considered as 1 hour. The 1000 hours of simulation results can be divided into n simulations of length m (such that $n \cdot m = 1000$ hours). Table II summarizes the different methods and simulation times used. For each method, the characteristic short-term extreme value is taken as the expected value of the short-term extreme distribution. Each method was used n_i times, providing n_i estimates of the expected short-term extreme. By fitting a normal distribution to these n_i estimates, the expected value and variance of each method can be estimated and compared [12]. For example, the APW_0.5 case described in Table II uses a Weibull fitting method on $m_i = 0.5$ hour simulations; with 1000 hours available, there are a total of $n_i = 2000$ realizations with which to characterize the performance of this method. On the other hand, the BMA_10 case uses a block maxima method in which ten simulation, each with a length of 1 hour ($m_i = 10$), are used, allowing for a total 100 realizations to be taken from the 1000 hours of available simulation time.

TABLE II. SUMMARY OF CASE-STUDY STATISTICAL METHODS FOR COMPARISON.

Case name	Method	Simulation duration (hr)	Number of realizations
APW_0.5	all-peaks Weibull	0.5	2000
APW_1	all-peaks Weibull	1	1000
APW_2	all-peaks Weibull	2	500
WTF_0.5	Weibull Tail-Fit	0.5	2000
WTF_1	Weibull Tail-Fit	1	1000
WTF_2	Weibull Tail-Fit	2	500
BMA_10	Block Maxima	10×1	100
BMA_20	Block Maxima	20×1	50
BMA_40	Block Maxima	40×1	25
POT_0.5	Peaks Over Threshold	0.5	2000
POT_1	Peaks Over Threshold	1	1000
POT_2	Peaks Over Threshold	2	500
EMP	Block Maxima	1	1000

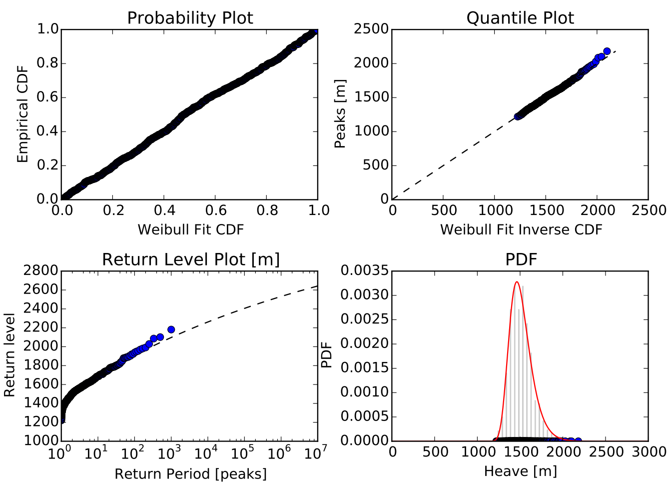

Fig. 2. Goodness of fit plots for GEV and all (1000 hours) 1-Hour extremes data.

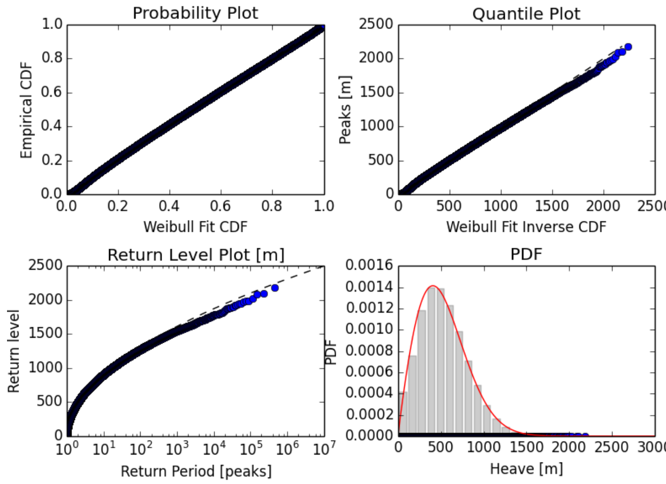

Fig. 3. Goodness of fit plots for Weibull and all (1000 hours) global peaks data.

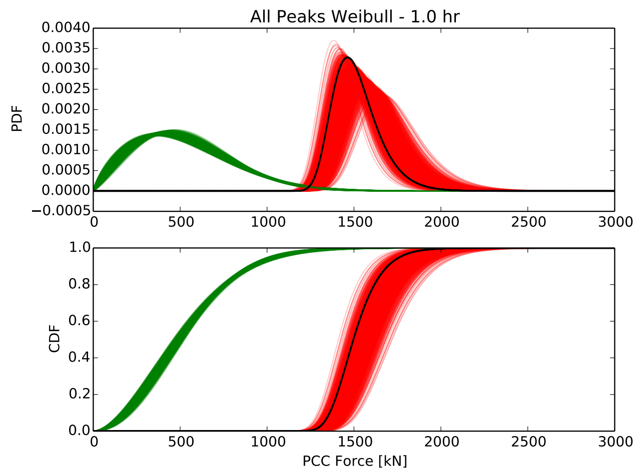

Fig. 4. Results from the 1-hour all-peaks Weibull method. Green - Weibull fit of global peaks. Red - Estimated distribution of 1-hour extremes. Black - True distribution of 1-hour extremes

IV. RESULTS

An empirical "truth" for this study was obtained by using the block maxima method with all available data. A GEV distribution was fit to the 1000 1-hour extremes. The resulting distribution has the following parameters: shape, $\xi = 0.0567$; scale, $\sigma = 1456$; location, $\mu = 112$. Since the shape parameter, ξ, is very close to zero, a Gumbel distribution would also be an acceptable option. Fig. 2 shows the goodness of fit with good agreement between the fitted GEV and the data.

Fig. 3 and 4 show an example of the analyses conducted for each case studied in the comparison. Here, the results for the 1-hour all-peaks Weibull case (APW_1) is considered. The goodness of fit plots in Fig. 3 show that the Weibull distribution is a good choice for the global peaks. Fig. 4 shows the fits from each of the 1000 realizations for this method. For this case, there are 1000 distinct Weibull fits of the peaks (green lines) and 1000 extrapolated 1-hour extreme distribution estimates (red lines). It can be seen that there is a large spread and that the method tends to be an overestimate compared to the empirical truth (black line). Additionally, Fig. 4 also illustrates the process of a peak distribution being transformed to an extreme distribution (this behavior, and its influence on the rational behind the Weibull tail-fit method was discussed in Section II-B).

The performance for each of the cases introduced in Table II is shown in Table III and Fig. 5. Fig. 5 shows the expected value and the 95% interval bounds of the normal distribution fitted to each of the method's expected value estimates. The horizontal lines are the expected value and 95% intervals of the "true" empirical distribution of 1-hour extremes. These same results are summarized in Table III with the simulation time required to execute each method.

For the block maxima methods, a Gumbel distribution was used instead of a GEV. When using the GEV with fewer than roughly 50 points, the spread was very large due to the fitted shape parameter being positive, negative, or zero for different realizations [8]. This results in three very different behaviors and causes large spread of predicted characteristic extreme values. Forcing the shape parameter to zero (therefore forcing the GEV to assume a Gumbel form) allows for reasonable fits

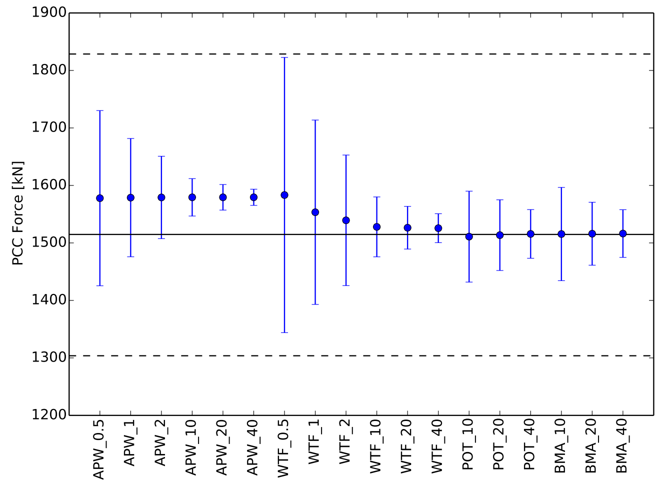

Fig. 5. Comparison of different methods. Black solid line is the expected value of the empirical "truth" 1-hour extreme distribution, black dashed lines are the bounds of the 95% interval. The expected values and 95% bounds from the different methods are shown in blue • and error bars.

TABLE III. SUMMARY IF STATISTICAL METHOD PERFORMANCE.

Case name	Expected value, EV [kN]	EV % difference	± 95% [kN]	Simulation time [hr]
APW_0.5	1577.8	-4.17%	304.8	0.5
APW_1	1578.8	-4.23%	205.8	1
APW_2	1579.1	-4.25%	143.3	2
APW_10	1579.3	-4.26%	65.1	10
APW_20	1579.3	-4.27%	44.5	20
APW_40	1579.3	-4.27%	28.2	40
WTF_0.5	1583.3	-4.53%	478.6	0.5
WTF_1	1553.4	-2.55%	320.7	1
WTF_2	1539.3	-1.62%	227.1	2
WTF_10	1527.9	-0.87%	104.2	10
WTF_20	1526.4	-0.77%	74.2	20
WTF_40	1525.6	-0.72%	50.3	40
BMA_10	1515.4	-0.05%	162.2	0.5
BMA_20	1516.0	-0.08%	109.5	1
BMA_40	1516.3	-0.10%	82.9	2
POT_10	1511.0	0.25%	158.2	10
POT_20	1513.5	0.08%	123.0	20
POT_40	1515.7	-0.06%	84.7	40
Empirical "truth"	**1514.7**			**1000**

with fewer points. The Generalized Pareto distribution used for the peaks-over-threshold method also shows very different behaviors depending on the sign of its shape parameter, and it was found that about 10 or more hours of simulation were needed to guarantee a reliable fit.

V. DISCUSSION

From Fig. 5, it is clear that the statistical methodology used to produce a short-term extreme response distribution can have a substantial influence on the results of an analysis. For the same simulation time, the all-peaks Weibull method provides lower variance than the Weibull tail-fit method, but does not, on average, predict the true expected extreme value with as much accuracy. To function properly, the peaks-over-threshold and block maxima methods require an order of magnitude more data than the all-peaks Weibull and Weibull tail-fit methods. On average, the peaks-over-threshold and block maxima methods predict the true expected value; however, a long simulation time is required to reduce the variance in these methods' predictions. For the purpose of this project it is likely a Weibull tail fit will be adopted as the method for obtaining the short term characteristic extreme value, since it seems to be the best compromise between accuracy and simulation time.

Future investigation into methods for obtaining the characteristic extreme value will focus on two aspects: (i) exploration of other, more novel methods, and (ii) consideration of a different characteristic extreme value. In this study, we used the expected value of each extreme distribution to serve as the characteristic extreme in the single point comparison between methods. However, there is ample reason to consider using a higher percentile in design applications. For instance, considering ocean wave amplitudes, Ochi points out that there is a 62.3% chance of the extreme wave exceeding the extreme distribution's (Rayleigh) modal value (i.e. peak of the distribution) [13]. Thus, many engineering applications employ some small risk parameter or probability of exceedance in selecting the percentile to indicate a single characteristic extreme. In fact, it may be wise to choose the probability of exceedance to use with a given distribution based on the that distribution's variability, as demonstrated by [14]. Here, the percentile used to determine a characteristic extreme is set based on the distribution's variance; thus, in cases with a large variability, a higher percentile is used.

It must be noted that the common use of a Weibull distribution to describe the response of floating structures is not completely theoretically justified, and is instead a useful and practical assumption [12]. In this particular study, because of the nature of the simple numerical model, which uses linear superposition of sinusoidal functions, and the narrow-banded nature of both the ocean spectrum and the WEC's dynamic system, the peaks of the response are expected to follow a Rayleigh distribution (see, e.g., [13]). Since the Rayleigh distribution is a subset of the Weibull distribution, the results using this method might be better in this particular case-study than with nonlinear numerical models. The goodness of fit plots in Fig. 3 show that a Weibull distribution is a good choice for the global peaks the data considered in this study.

VI. CONCLUSION

A series of short-term extreme response methods have been presented and compared. Each method was implemented using datasets of a range of sizes. The results of this analysis show that these statistical analysis methods and the results that they produce must be used with care and an understanding of their performance trade-offs. While none of the considered methods is clearly dominant above the others, for applications in which the model requires some significant amount of time to run, the Weibull tail-fitting approach appears to be a good a balance between accuracy, low variance and efficient usage of data. Future work will focus on investigating more novel methods and on considering different characteristic extreme values to represent the distribution.

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