Aeroelastic Stability Analysis of a Darrieus Wind Turbine

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Abstract

An aeroelastic stability analysis has been developed for predicting flutter instabilities on vertical axis wind turbines. This report describes the analytical model and mathematical formulation of the problem as well as the physical mechanism that creates flutter in Darrieus turbines. Theoretical results are compared with measured experimental data from flutter tests of the Sandia 2 Meter turbine. Based on this comparison, the analysis appears to be an adequate design evaluation tool.

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Introduction

Tests of a scale model Darrieus wind turbine have shown that under certain conditions, the turbine may experience a flutter instability. Although flutter has not been observed on a full scale turbine, an analysis is required that will guarantee the stability of a future design.

A survey was made of the currently available aeroelastic stability analyses which are listed in Refs. 1-5. The majority of these analyses are complex, exact treatments of the aeroelastic problem. Although these analyses will probably yield accurate results, they are cumbersome to implement and the physical understanding of the flutter problem is easily lost. Ref. 2 is a simplified method which revealed that mass balance of the blade cross section was not important for flutter stability; a finding which has had tremendous impact on blade manufacturing costs.

In this report, an approach was chosen that can give physical insight as well as reasonable numerical results. Test data from the Sandia 2 Meter turbine proved invaluable in guiding the development of the analysis along a path that would yield a maximum of understanding with a minimum amount of mathematical complexity.

Mathematical Formulation of the Flutter Problem

The modal analysis method was used as the basis for the flutter analysis. This approach is used frequently for
analysis of helicopter blade response and was used by Ref. 1 for the vertical axis wind turbine. This method allows the analyst the freedom to describe the structural characteristics of the turbine in minute detail by using NA STRAN finite element modeling techniques to generate the necessary natural frequencies and mode shapes. The general equations of motion for the aeroelastic analysis of the turbine are as follows:

$$[M]\ddot{\{x\}} + [D]\dot{\{x\}} + [K]\{x\} = -\Omega^2 [KM]\{x\} - 2\Omega [C]\dot{\{x\}} + [AK]\{x\}$$

$$+ [AD]\ddot{x} + [AM]\{x\} + \{F(t)\}$$

where

- \{X\} = structural displacement response
- [M] = structural mass matrix
- [D] = structural damping matrix (diagonal matrix)
- [K] = structural stiffness matrix including centrifugal stiffening
- [KM] = centrifugal softening matrix
- [C] = Coriolis matrix
- [AK] = aerodynamic stiffness matrix
- [AD] = aerodynamic damping matrix
- [AM] = aerodynamic mass matrix
- \(\Omega\) = turbine rotational speed
- \{F(t)\} = time dependent external forces

The forces \(F(t)\) consist of harmonic aerodynamic forcing functions which are independent of the structural response. These forces do not affect the stability of the turbine and are therefore neglected.
To utilize modal analysis techniques, the solution to Eq. 1 is taken to be a linear combination of the normal modes of the following equation:

\[ [M][\dot{x}] + [K][x] = 0 \]  

(2)

This equation represents the structural properties of the turbine and is solved by finite element techniques using the NASTRAN Code. The normal modes contain geometric stiffness effects resulting from centrifugal loading.

If \( \phi_i \) are the mode shapes of Eq. 2, these modes can be assembled into the modal matrix \([\phi]\), and the physical response \([x]\) related to the modal response \([q]\) as follows:

\[ [x] = [\phi][q] \]

Substituting this equation into Eq. 1 and then premultiplying by \([\phi]^T\) yields:

\[
\begin{align*}
[GI]\{\ddot{q}\} + [2\xi\omega_N GI]\{\dot{q}\} + [GI\omega_N^2]\{q\} &= -2\Omega [\phi]^T[C][\phi]\{\dot{q}\} - \Omega^2 [\phi]^T[KM][\phi]\{q\} \\
&+ [\phi]^T[AK][\phi]\{q\} + [\phi]^T[AD][\phi]\{\dot{q}\} \\
&+ [\phi]^T[AM][\phi]\{\ddot{q}\}
\end{align*}
\]

(3)

where:

\([q]\) = modal response coordinates

\([GI]\) = diagonal generalized mass matrix

\([2\xi\omega_N GI]\) = diagonal modal damping matrix

\([GI\omega_N^2]\) = diagonal generalized stiffness matrix

\(\xi\) = structural damping factor

\(\omega_N\) = natural frequency

\(\Omega\) = turbine RPM
For very small values of structural damping or for proportional damping, the left hand side of Eq. 3 is completely decoupled as discussed in Ref. 6. Since all terms in Eq. 3 are functions of the modal response coordinates, the right hand side can be combined with the left hand side yielding:

\[
[GI - AMB]{\ddot{q}} + [2\xi\omega_N GI + CB - ADB]{\dot{q}} + [GI\omega_N^2 - AKB + KMB]{q} = 0 \tag{4}
\]

where:

\[
AMB = [\phi]^T[AM][\phi]
\]

\[
ADB = [\phi]^T[AD][\phi]
\]

\[
AKB = [\phi]^T[AK][\phi]
\]

\[
CB = 2\alpha[\phi]^T[C][\phi]
\]

\[
KMB = \omega^2[\phi]^T[KM][\phi]
\]

Standard eigenvalue routines are used to solve this system of equations for the complex eigenvalues, \( \lambda = \sigma + iw \). The real part of the eigenvalue, \( \sigma \), determines the stability of the mode. A positive value indicates an instability and a negative value indicates a stable configuration. The imaginary component, \( iw \), contains the flutter frequency.

Description of the Flutter Model

A typical NASTRAN model for the flutter analysis is shown in Fig. 1. A single blade is used with a tower that has the proportional torsional stiffness for that blade. The drive train is modeled by a torsional spring which represents the low speed shaft stiffness. The troposkein shaped blade is represented by a series of straight beam elements. To simplify
the calculation of aerodynamic forces, each beam element contains an additional intermediate node as shown in Fig. 1. The distributed aerodynamic loads are computed for the beam element and these forces are lumped to the intermediate node which also contains the concentrated mass properties of the beam element. The cross section of the turbine blade in Fig. 1 shows the relative location of the midchord, elastic axis, and center of mass. The flutter instability involves an interaction between the two modes shown in Fig. 1. These two modes do not include tower translation, so the top and bottom of the tower are pin jointed.

Derivation of the Equations of Motion

The equations of motion for the wind turbine are derived from Newton's laws using the following conditions:

1. All equations are expressed in the rotating coordinate system. This eliminates time-dependent coefficients that are present if the equations are derived in the fixed coordinate system.

2. The turbine blade dynamics are based on concentrated mass particles connected by massless elastic beams. Fig. 2 illustrates the coordinate systems and degrees of freedom needed to define the blade motion. The "R" coordinate system represents the rotating coordinate system aligned with the undeformed tower and blades. For convenience, at each mass particle on the blade, a local "l" coordinate system is defined
parallel to the "R" system. The "2" coordinate system is aligned with the local curvature of the blade, and is related to the "1" coordinate system by the angle $\gamma$ as follows:

\[
\begin{bmatrix}
  i_2 \\
  j_2 \\
  k_2
\end{bmatrix} =
\begin{bmatrix}
  \cos \gamma & 0 & -\sin \gamma \\
  0 & 1 & 0 \\
  \sin \gamma & 0 & \cos \gamma
\end{bmatrix}
\begin{bmatrix}
  i_1 \\
  j_1 \\
  k_1
\end{bmatrix}
\]  

(5)

Coordinate system "3" follows the blade section during elastic deformations. The relation between the "3" coordinate system and the "2" coordinate system is given as:

\[
\begin{bmatrix}
  i_3 \\
  j_3 \\
  k_3
\end{bmatrix} =
\begin{bmatrix}
  1 & \phi_z & -\phi_y \\
  -\phi_z & 1 & \phi_x \\
  \phi_y & -\phi_x & 1
\end{bmatrix}
\begin{bmatrix}
  i_2 \\
  j_2 \\
  k_2
\end{bmatrix}
\]  

(6)

The rotations $\phi_x$, $\phi_y$ and $\phi_z$ are transformed to the "R" coordinate system as shown below:

\[
\begin{bmatrix}
  \phi_{xr} \\
  \phi_{yr} \\
  \phi_{zr}
\end{bmatrix} =
\begin{bmatrix}
  \cos \gamma & 0 & \sin \gamma \\
  0 & 1 & 0 \\
  -\sin \gamma & 0 & \cos \gamma
\end{bmatrix}
\begin{bmatrix}
  \phi_x \\
  \phi_y \\
  \phi_z
\end{bmatrix}
\]

where $\phi_{xr}$, $\phi_{yr}$, and $\phi_{zr}$ are the rotations about the $i_r$, $j_r$, and $k_r$ axes.

Referring to Fig. 3, the equations for dynamic equilibrium of a elemental blade section can be written. In the limiting case of an infinitesimal blade section length, a point mass results with the following equations of motion:

\[
\vec{F}_1 - \vec{F}_2 = \vec{m}\vec{a} - \vec{F}_{aero} \quad \text{(Force Equilibrium)}
\]

\[
\vec{M}_1 - \vec{M}_2 = \vec{p} + \vec{e}_{gr}\vec{x}\vec{ma} - \vec{M}_{AERO} \quad \text{(Moment Equilibrium)}
\]

(7)  

(8)
The underlined components of the above equation are due to the dynamics of blade motion and are derived below.

For a rotating dynamic system, the position, velocity and acceleration of a particle of mass is given by:

\[ \ddot{\mathbf{r}} = \dot{\mathbf{r}}_p \]
\[ \dot{\mathbf{r}} = \mathbf{r}_p + \omega \mathbf{x}_p \]
\[ \mathbf{r} = \mathbf{r}_p + \omega \mathbf{x}_p + 2\omega \mathbf{x}_p + \omega \mathbf{x} \mathbf{x}_p \]

where \( \mathbf{r}_p \) is the position vector of the concentrated mass and \( \omega \) is the angular velocity of the rotating coordinate system.

Referring to Figure 2, the position vector of the center of mass of a blade section is:

\[ \mathbf{r} = (r_0 + U_x) \mathbf{i} + U_y \mathbf{j} + U_z \mathbf{k} + \mathbf{e}g \mathbf{j}_3 \]

The angular velocity of the blade section is

\[ \omega_3 = \dot{\phi}_x \mathbf{i}_3 + \dot{\phi}_y \mathbf{j}_3 + \dot{\phi}_z \mathbf{k}_3 + \Omega \mathbf{k}_3 \]

Transforming the angular velocity to the "3" coordinate system yields,

\[ \omega_3 = (\dot{\phi}_x - \Omega \sin \gamma - \phi_y \Omega \cos \gamma) \mathbf{i}_3 + (\phi_z \Omega \sin \gamma + \dot{\phi}_y + \phi_x \Omega \cos \gamma) \mathbf{j}_3 + (-\Omega \phi_y \sin \gamma + \dot{\phi}_z + \Omega \cos \gamma) \mathbf{k}_3 \]

Beginning with Eq. 9 in coordinate system "3" and using Eqs. 5 and 6 to transform to the "R" coordinate system, the particle velocity expressed in the "R" system is:

\[ \mathbf{v} = \mathbf{U} = [\dot{U}_x - \Omega U_y - \mathbf{e}g(\Omega + \dot{\phi}_z)] \mathbf{i}_r + [\dot{U}_y + \Omega (r_0 + U_x) - \mathbf{e}g \phi_z] \mathbf{j}_r + [\dot{U}_z + \mathbf{e}g \dot{\phi}_x] \mathbf{k}_r \]
The acceleration of the particle in the "R" system is given by:

\[ \ddot{a} = \ddot{r} = [\ddot{u}_x - 2\Omega \dot{u}_y - (r_0 + u_x)\Omega^2 + (\Omega^2 \phi_{zr} - \dot{\phi}_{zr}) eg] i_r \\
+ [\ddot{u}_y + 2\Omega \dot{u}_x - \Omega^2 u_y - 2\dot{\phi}_{zr} eg - \Omega^2 eg] j_r \\
+ [\ddot{u}_z + \dot{\phi}_{xz} eg] k_r \]

The inertia force, \( \ddot{m}a \), is written in matrix form as follows:

\[
\ddot{m}a = \begin{bmatrix}
m & 0 & 0 & 0 & 0 & -meg \\
0 & m & 0 & 0 & 0 & 0 \\
0 & 0 & m & meg & 0 & 0 \\
\end{bmatrix}
\begin{pmatrix}
\ddot{u}_x \\
\ddot{u}_y \\
\ddot{u}_z \\
\end{pmatrix} + \begin{bmatrix}
0 & -2m\Omega & 0 & 0 & 0 & 0 \\
2m\Omega & 0 & 0 & 0 & 0 & -2meg\Omega \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{pmatrix}
\dot{u}_x \\
\dot{u}_y \\
\dot{u}_z \\
\dot{\phi}_{xr} \\
\dot{\phi}_{yr} \\
\dot{\phi}_{zr} \\
\end{pmatrix} = \begin{bmatrix}
\ddot{u}_x \\
\ddot{u}_y \\
\ddot{u}_z \\
\ddot{\phi}_{xr} \\
\ddot{\phi}_{yr} \\
\ddot{\phi}_{zr} \\
\end{pmatrix}
\]

The angular momentum of the blade section is given by:

\[ [P_3] = [I_3] \{\omega_3\} \]

12
where \([I_3]\) is the inertia matrix for the blade section and \(\{\omega_3\}\) is the angular velocity of the section. The rate of change of angular momentum is given by:

\[
\dot{P} = \dot{\{P_3\}} + \{\omega_3\} \times \{P_3\}
\] (16)

The inertia matrix for a blade section is expressed in the blade coordinate system as:

\[
[I_3] = \begin{bmatrix}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{bmatrix}
\]

The products of inertia are assumed to be small and have been neglected to simplify the equations. Substituting Eq. 11 into Eq. 16 and transforming to the "R" coordinate system results in the following equations in matrix form:
\[ \mathbf{\ddot{p}} = \begin{bmatrix} 0 & 0 & 0 & \sin \gamma (I_{zz} - I_{xx}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sin \gamma (I_{zz} - I_{xx}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \sin \gamma (I_{zz} - I_{xx}) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{u}_x \\ \ddot{u}_y \\ \ddot{u}_z \\ \ddot{\phi}_x \\ \ddot{\phi}_y \\ \ddot{\phi}_z \end{bmatrix} \]

\[ \begin{bmatrix} 0 & 0 & 0 & \sin \gamma (I_{zz} - I_{xx}) + I_{yy} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

(17)
The center of mass offset from the elastic axis creates a moment about the elastic axis. The mass offset expressed in the "R" coordinate system is:

\[ \bar{e}_r = e_g \left[ (-\phi_z \cos \gamma + \phi_x \sin \gamma)i_r + j_r + (\phi_z \sin \gamma + \phi_x \cos \gamma)k_r \right] \tag{18} \]

The moment due to the mass offset is:

\[ \bar{e}_r \times \bar{m} = \begin{bmatrix} i_r & j_r & k_r \\ e_g(-\phi_z \cos \gamma + \phi_x \sin \gamma) & e_g & e_g(\phi_z \sin \gamma + \phi_x \cos \gamma) \\ \bar{m}_x & \bar{m}_y & \bar{m}_z \end{bmatrix} \tag{19} \]

Substituting Eq. 14 into Eq. 19 yields:

\[ \bar{e}_r \times \bar{m} = \begin{bmatrix} 0 & 0 & \text{meg} & \text{meg}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\text{meg} & 0 & 0 & 0 & 0 & \text{meg}^2 \end{bmatrix} \begin{bmatrix} \ddot{u}_x \\ \ddot{u}_y \\ \ddot{u}_z \\ \ddot{\phi}_{xr} \\ \ddot{\phi}_{yr} \\ \ddot{\phi}_{zr} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \text{mn}^2 \text{eg}^2 & -\text{mr}_o \text{n}^2 \text{eg} & 0 \\ 0 & 0 & 0 & -\text{mr}_o \text{n}^2 \text{eg} & 0 & 0 \\ \text{mn}^2 \text{eg} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{u}_x \\ \dot{u}_y \\ \dot{u}_z \\ \dot{\phi}_{xr} \\ \dot{\phi}_{yr} \\ \dot{\phi}_{zr} \end{bmatrix} \tag{20} \]

The complete equations of motion are assembled from equations 14, 17, and 20 and are written as follows:
\[
\begin{align*}
\mathbf{F}_{XR} &= \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & m & \text{meg} \\ 0 & 0 & 0 & \text{meg} \end{bmatrix}, \\
\mathbf{F}_{YR} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
\mathbf{M}_{XR} &= \begin{bmatrix} 0 & 0 & \text{meg} & \text{meg}^2 + I_{xx}\cos^2\gamma + I_{zz}\sin^2\gamma \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{yy} \\ -\text{meg} & 0 & 0 & \text{sin}\gamma\cos\gamma(I_{zz} - I_{xx}) \end{bmatrix}, \\
\mathbf{M}_{YR} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
\mathbf{M}_{ZR} &= \begin{bmatrix} 0 & 0 & \text{sin}\gamma\cos\gamma(I_{zz} - I_{xx}) & 0 \\ 0 & 0 & 0 & \text{meg}^2 + I_{xx}\sin^2\gamma + I_{zz}\cos^2\gamma \\ -\text{meg} & 0 & 0 & \text{sin}\gamma\cos\gamma(I_{zz} - I_{xx}) \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\end{align*}
\]
The first portion of Eq. 21 is the mass matrix for the blade section, which is represented by \([M]\) in Eq. 1. The second matrix in Eq. 21 is the Coriolis matrix, which appears as \([C]\) in Eq. 1. The last portion of Eq. 21 is the centrifugal softening matrix, or \([KM]\) in Eq. 1.

For most turbine analyses, the blade inertias \(I_{XX}, I_{YY}, I_{ZZ}\) are not included in the NASTRAN model and are therefore not used in the flutter analysis. However, they are included in this derivation for completeness.

**Derivation of Aerodynamic Loads**

The aerodynamic loads acting on the turbine blades are derived using unsteady aerodynamic theory as discussed in Refs. 7-10. These references cite Theodorsen's original work on a two-dimensional airfoil oscillating in a steady air stream. This theory accounts for all possible motions of the blade section that will produce aerodynamic loads.

Several simplifying assumptions, listed below, are made for the airload calculations:

1) Two-dimensional strip theory assumed applicable
2) No stall considered
3) Chord line and zero-lift line assumed to coincide
4) No inflow permitted through the turbine
5) Blade relative wind velocity assumed constant during turbine rotation
6) Aerodynamic center located at quarter chord point
7) Turbine wake not modeled

8) Small angles assumed, linearized aerodynamic equations

As outlined in Ref. 10, the unsteady lift, moment, and drag acting on a blade section are given by the following equations:

\[ L = \frac{ao}{2} \rho b V^2 \left\{ CK \left[ -\frac{2h}{V} + (1 - 2a) \frac{b^2}{V^2} \frac{\dot{\theta}}{V} + 2\theta \right] - \frac{b^2}{V^2} \frac{\ddot{h}}{V} - \frac{b^2 a \dot{\theta}}{V^2} + \frac{b}{V} \ddot{\theta} \right\} \quad (22) \]

\[ M = \frac{-ao}{2} \rho b^2 V^2 \left\{ \frac{ab}{V^2} \dddot{h} + (1 + 2a) \frac{1}{V} CK \dot{h} + \left( \frac{1}{8} + a^2 \right) \frac{b^2}{V^2} \ddot{\theta} \right. \]
\[ \left. - \left[ a - \frac{3}{2} + 2 \left( \frac{3}{8} - a^2 \right) CK \right] \frac{b}{V} \ddot{\theta} - (1 + 2a) CK \dot{\theta} \right\} \quad (23) \]

\[ D = \frac{1}{2} \rho V^2 c C_{do} \quad (24) \]

where:

- \( a \) = nondimensional distance from elastic axis to midchord (positive if elastic axis is aft of midchord, expressed as a fraction of \( b \))
- \( ao \) = lift curve slope
- \( b \) = 1/2 chord length
- \( c \) = chord length
- \( C_{do} \) = drag coefficient for airfoil
- \( CK \) = Theodorsen's lift deficiency function
- \( D \) = drag of airfoil (per unit span)
- \( h \) = vertical translation of the airfoil at elastic axis (positive up)
- \( L \) = unsteady lift at elastic axis (per unit span)
- \( M \) = unsteady moment at elastic axis (per unit span)
- \( V \) = relative wind velocity for blade section, \( \approx V_0 + \Omega r_o \)
- \( V_0 \) = wind velocity
- \( \dot{\theta} \) = pitch rotation of the airfoil (positive nose-up)
- \( \rho \) = air density
The expressions for $\dot{h}$ and $\ddot{h}$ are obtained from Eqs. 12 and 13 and are then resolved into the "2" coordinate system to give:

$$\dot{h} = (\dot{U}_x \sin\gamma - \Omega \dot{U}_y \sin\gamma + \dot{U}_z \cos\gamma) k_2$$

$$\ddot{h} = (\ddot{U}_x \sin\gamma - 2\Omega \dot{U}_y \sin\gamma - (r_o + U_x) \Omega^2 \sin\gamma + \ddot{U}_z \cos\gamma) k_2$$

(25)

The geometric pitch of the airfoil is given by the following equations:

$$\theta = (\phi_{xr} \cos\gamma - \phi_{zr} \sin\gamma)i_2$$

$$\dot{\theta} = (\dot{\phi}_{xr} \cos\gamma - \dot{\phi}_{zr} \sin\gamma)i_2$$

$$\ddot{\theta} = (\ddot{\phi}_{xr} \cos\gamma - \ddot{\phi}_{zr} \sin\gamma)i_2$$

(26)

Eqs. 25 and 26 are substituted into Eqs. 22 and 23 to give the lift and moment acting on the blade section. In Eq. 22, the lift can be separated into two components. The first component is the circulatory lift, $L_c$, which depends on the value of CK. This component of the lift vector acts perpendicular to the relative wind and is the result of circulation produced by the lifting airfoil. The second component is the noncirculatory lift, $L_{NC}$, which includes the "apparent mass" of the air stream and acts perpendicular to the chord line of the airfoil.

Figure 4 shows the aerodynamic loads acting on a typical blade cross section. The lift, drag, and moment can be resolved into the blade coordinate system which yields:
\[ F_z = (L_c + D\alpha + L_{NC})k_3 \]
\[ F_y = (L_c\alpha - D)j_3 \]
\[ M_x = M_i j_3 \]  \hspace{1cm} (27)

The angle of attack, \( \alpha \), is composed of the geometric pitch angle, \( \phi \), plus the induced angle of attack, \( \beta \). The induced angle of attack is the angle due to blade motion, and is the ratio of the blade vertical velocity to the horizontal velocity. The equation for \( \alpha \) is as follows:

\[
\alpha = \phi_x \cos \gamma - \phi_z \sin \gamma - \left( U_x \sin \gamma - \omega U_y \sin \gamma + U_z \cos \gamma \right) / V \]  \hspace{1cm} (28)

Substituting Eqs. 22, 23, 24, and 28 into Eq. 27 yields the aerodynamic forces in the blade coordinate system. These forces are then resolved into the "R" coordinate system by using Eqs. 5 and 6.

The final expression for the aerodynamic loads are written in matrix form as follows:
\[
\begin{align*}
\mathbf{F}_x &= \begin{bmatrix}
-A_1b^2S_y^2 & 0 & 0 & 0 \\
0 & -A_1b^2S_y^2 & 0 & 0 \\
0 & 0 & -A_1b^2S_y^2 & 0 \\
0 & 0 & 0 & -A_1b^2S_y^2 \\
\end{bmatrix}
\end{align*}
\]

\[
\mathbf{F}_y = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\mathbf{F}_{z} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{align*}
\mathbf{M}_x &= \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\end{align*}
\]

\[
\mathbf{M}_y = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{align*}
\mathbf{M}_{z} &= \begin{bmatrix}
-A_1b^2S_y^2 & 0 & 0 & 0 \\
0 & -A_1b^2S_y^2 & 0 & 0 \\
0 & 0 & -A_1b^2S_y^2 & 0 \\
0 & 0 & 0 & -A_1b^2S_y^2 \\
\end{bmatrix}
\end{align*}
\]
where:

\[ A_1 = \rho ba_0 \Delta L / 2. \]
\[ A_3 = 1/2 \rho V^2 b C_d \Delta L \]
\[ A_4 = A_1 (1 + 2a) V C K b \]
\[ A_5 = A_1 [a - 1/2 + 2(1/4 - a^2) C K] \]
\[ A_6 = A_1 (1/8 + a^2) b^2 \]
\[ A_7 = A_1 C K (1 - 2a) b V \]
\[ S_\gamma = \sin \gamma \]
\[ C_\gamma = \cos \gamma \]

The first matrix in Eq. 29 is the aerodynamic mass matrix corresponding to [AM] in Eq. 1. The second matrix is the aerodynamic damping matrix which is represented by [AD] in Eq. 1. The last portion is the aerodynamic stiffness matrix denoted by [AK]. Note that neither [AD] nor [AK] are symmetric because the aerodynamic forces do not constitute a conservative system.

The aerodynamic loads in Eq. 29 include the Theodorsen Function, CK, which is a measure of the unsteadiness of the flow field. The parameter, CK, is a complex number which alters the phase angle between the airfoil oscillation and the resultant aerodynamic forces. Its value is dependent on the reduced frequency, or Strouhal number. For the flutter instabilities observed on the VAWT, the Strouhal number is very low which renders CK equal to unity. This corresponds to a "quasi-steady" flow field which appears to be an adequate representation of the aerodynamic loads for the VAWT flutter problem.
Flutter Test Program

To verify the accuracy of the flutter analysis, a flutter test program was conducted. This test program utilized the Sandia 2 Meter turbine with several sets of aluminum blades. These test were conducted in the following manner:

1. The non-rotating blade frequencies were measured for comparison with the NASTRAN model of the turbine. The modal damping of the turbine was also measured but it was difficult to get a repeatable value. Generally, the modal damping varied from .1% to .4% critical. A value of .35% was used in the analysis.

2. The turbine speed was set and an impulse was applied to the turbine through the brake system to trigger the flutter instability. A torque meter was used to record the resulting torque oscillation which increased during a flutter instability and decayed to a stable configuration. The mechanism that creates flutter is clearly shown in slow motion films of the flutter instability. The instability is due to coupling between the flatwise bending and torsion mode shown in Fig. 1. The flatwise bending mode involves radial motion of the blade which creates Coriolis forces that amplify the response of the torsion mode. The resulting elastic deflections cause changes in the aerodynamic forces which add energy to the vibrating blade and create flutter.
Correlation of Theory and Experiment

The flutter analysis was verified by comparing the theoretical results with measured test data. Fig. 5 shows the flutter stability for the two meter turbine with three aluminum blades (NACA 0012, CHORD = 2.91 in) and a truss tower. This figure shows the variation in modal damping and modal frequency with turbine speed. The modal damping curve was calculated with zero structural damping. This damping curve is very shallow which means that small changes in the structural damping have a large influence on the flutter speed. The theoretical flutter speed was found by adding .35% structural damping to the modal damping curve, giving 850 RPM as the flutter speed. This is in fair agreement with the measured flutter speed of 745 RPM. The calculated flutter frequency, 18.5 Hz, agrees well with the measured flutter frequency, 18 Hz.

The turbine was modified by replacing the truss tower with a torsionally stiff pipe tower. The flutter instability did not occur in tests up to 900 RPM. Theoretical results agree with this data.

Wind speed effects were evaluated by testing the turbine in 25 mph winds. The turbine configuration consisted of three aluminum blades (NACA 0012, CHORD = 2.91 in) and a truss tower. Fig. 6 shows the measured flutter speed to be in the range of 705-720 RPM at a flutter frequency of 18 Hz. The theoretical results indicate flutter at 695 RPM at a frequency of 16.5 Hz. The theory predicts that the wind velocity has a
larger influence on the flutter speed than the test results indicate. This is probably due to the simplifying assumptions in the aerodynamic load calculations.

A larger set of blades (NACA 0012, CHORD = 3.47 in) was installed on the two meter turbine with the truss tower as shown in Fig. 7. This set of blades did not flutter up to speeds of 1050 RPM. The analysis, however, predicts flutter at 1000 RPM. This disagrees with the test results, but since flutter was not observed in the test, the magnitude of the theoretical error is unknown.

Fig. 8 displays the flutter results for the small blades (NACA 0015, CHORD = 2.31 in). Flutter was noted at 777 RPM at a frequency of 18.5 Hz. Theoretically the flutter speed is 865 RPM at a frequency of 18.7 Hz, which correlates well with the test data.

The stability of the 17 Meter Sandia turbine is shown in Fig. 9. The calculated flutter speed is 176 RPM at 4.5 Hz, which is well above the 50 RPM operating speed.

These results indicate that the analysis is capable of assessing the effect of turbine design changes on flutter speed. The stability trends are predicted accurately by the program and the numerical results are sufficiently accurate to establish confidence in the analysis.
Conclusions

The analysis developed here has shown to be a useful tool for understanding and predicting flutter of Darrieus, vertical axis wind turbines. Additional test data is needed to fully verify the accuracy of the analysis and to establish error bounds. The analysis is potentially weak in the areas of aerodynamic force calculation since several simplifying assumptions have been made. A more complex aerodynamic model would improve the predictive capability of the model.

Several observations have been made based on the results of the analysis and tests:

1. Flutter is a result of aeroelastic coupling of two primary blade modes; the first flatwise bending mode and the first torsion mode.

2. Flutter does not require that two blade modes be in resonance. The frequencies of the flatwise mode and the torsion mode do not converge in the operating RPM range. However, increasing the separation of the flatwise mode and the torsion mode increases the flutter RPM.

3. Tower and drive train torsional stiffness affect the torsion mode frequency which affects the flutter RPM.

4. Flutter occurs at a frequency very near the flatwise mode frequency.

5. Wind velocity reduces the flutter RPM, but for operational wind speeds, the effect is relatively small.
Recommendations for Future Work

The flutter problem should be investigated further with a comprehensive program of testing and theoretical investigations. It is recommended that the following problem areas be addressed:

1. Determine the effect of chordwise mass balance on the flutter stability. This will involve the fabrication of blades with significant chordwise center of gravity offset. Ref. 2 predicts that mass offset has little influence on flutter stability. This should be verified with test data and results from the analysis.

2. Study the effect of blade frequency placement and the role of tower torsional stiffness on flutter stability.

3. Evaluate the effect of wind velocity on flutter stability in greater detail. Very high, short duration wind gusts may reduce the flutter RPM.

4. Modify the aerodynamic load model to account for stall, dynamic inflow, turbine wake effects, and periodic variation in relative wind velocity.

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Figure 1. Mathematical Modeling Technique for the Flutter Analysis
Figure 2. Coordinate Systems and Degrees of Freedom used in the Flutter Analysis
\( \bar{M}_{\text{AERO}} \) = AERODYNAMIC MOMENT ABOUT ELASTIC AXIS

\( \bar{F}_{\text{AERO}} \) = AERODYNAMIC FORCE AT ELASTIC AXIS

\( m\ddot{a} \) = INERTIA FORCE AT CENTER OF MASS

\( \dot{\phi} \) = RATE OF CHANGE OF ANGULAR MOMENTUM VECTOR

\( \bar{M}_1, \bar{M}_2 \) = MOMENT RESULTANTS AT INBOARD AND OUTBOARD ENDS OF SEGMENT

\( \bar{F}_1, \bar{F}_2 \) = FORCE RESULTANTS AT INBOARD AND OUTBOARD ENDS OF SEGMENT

Figure 3. Force and Moment Equilibrium for a Blade Element
Figure 4. Aerodynamic Forces and Moments Acting on an Airfoil Section
FLUTTER RESULTS
2 METER TURBINE
3 ALUMINUM BLADES
CHORD = 2.91 in.,
NACA 0012
TRUSS TOWER
WIND SPEED = 0
● EXPERIMENTAL DATA
■ THEORY

Figure 5. Comparison of Calculated and Measured Flutter Stability for the 2 Meter Turbine (NACA 0012, CHORD = 2.91 in)
Figure 6. Effect of Wind Speed on Flutter Stability of the 2 Meter Turbine (NACA 0012, CHORD = 2.91 in)
Figure 7. Comparison of Calculated and Measured Flutter Stability for the 2 Meter Turbine (NACA 0012, CHORD = 3.47 in)
Figure 8. Comparison of Calculated and Measured Flutter Stability for the 2 Meter Turbine (NACA 0015, CHORD - 2.31 in)
Figure 9. Flutter Stability Calculations for the 17 Meter Sandia Turbine
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