Estimation of Parameters for Single Diode Models Using Measured IV Curves

Clifford W. Hansen

Sandia National Laboratories, Albuquerque, NM, 87185-1033, USA

ABSTRACT — **Many popular models for photovoltaic system performance (e.g., [1], [2]) employ a single diode model (e.g., [3]) to compute the IV curve for a module or string of modules for given irradiance and temperature conditions. Most commonly (e.g., [4]), parameters are determined using only current and voltage at short circuit, open circuit and maximum power from a single IV curve at standard test conditions, along with reported temperature coefficients. In contrast, module testing frequently records IV curves at a wide range of irradiance and temperature conditions, such as those specified in IEC 61853-1 [5], which, when available, should also be used to parameterize the performance model. We propose a parameter estimation method that makes use of the full range of available IV curves, and demonstrate the accuracy of the resulting performance model.**

Index Terms **— semiconductor device modeling, photovoltaic systems, parameter estimation.**

I. INTRODUCTION

A single diode model is a popular way to represent the electrical performance of a photovoltaic (PV) module. A single diode model is formulated by extending the ideal diode law to account for parasitic series and shunt resistances ([3]) and by adding equations that describe how model terms (e.g., photocurrent) vary with irradiance and cell temperature. Various single diode models exist (e.g., [1], [2]), and some implementations are in wide commercial use (e.g., [6]).

The primary challenge to the use of a single diode model is the determination of values for its parameters. Many parameter estimation techniques have been proposed (see Sect. 3 for a brief overview). A successful method for estimating parameters from measured IV curve data should obtain all values necessary to perform model calculations. The method's results should be reproducible, i.e., the same parameter values should be obtained each time a particular data set is examined, and robust to measurement error. Moreover, the parameter estimation method should honor constraints on parameter values arising from physical meaning (e.g., resistance should be positive) or from model formulation (e.g., the diode ideality factor should have the same value for every IV curve).

We present here a method for obtaining parameters for a single diode model [2] of a PV module that attempts to meet these criteria. Our method relies on, and uses, data from measured IV curves over a range of irradiance and temperature conditions. Moreover, our method imposes constraints on parameter values so that parameter values are physically meaningful and are consistent with the model's assumptions.

We first outline the single diode model that serves to illustrate our proposed method (Sect. II). We review available parameter estimation methods in Sect. III. Our method for parameter estimation is described in Sect. IV; we present results from applying our method to measured IV curves in Sect. V.

II. SINGLE DIODE PERFORMANCE MODEL

The single diode model for a solar cell (e.g., [2], Eq. 1) can be derived from physical principles (e.g., [3]) and is often interpreted by an equivalent circuit comprising a current source, a diode, a parallel resistor and a series resistor. For a module comprising N_S identical cells in series, the IV characteristic is expressed as:

$$I = I_L - I_O\left[\exp\left(\frac{V + IR_s}{N_S n V_{th}}\right) - 1\right] - \frac{V + IR_s}{R_{sh}} \qquad (1)$$

where n is the diode ideality factor and $V_{th} = kT_C/q$ is termed the thermal voltage (V), which is determined from cell temperature T_C (K), Boltzmann's constant k (eV/K) and the elementary charge q (C). The parameters I_L, I_O, R_S, R_{sh}, and n are commonly referred as "the five parameters" from which the term "five parameter model" originates. In this presentation, values for the series resistance R_S and shunt resistance R_{sh} are expressed at the module level; values at the cell level can be obtained by dividing the module values by N_S (e.g., [7]).

Eq. (1) cannot be solved for current (or voltage) explicitly using elementary functions; however, current can be expressed as a function of voltage $I = I(V)$ (or $V = V(I)$) by using the transcendental Lambert's W function [8] as presented by several authors ([9], [10]). Lambert's W function is the solution $W(x)$ of the equation $x = W(x)\exp[W(x)]$.

Eq. (1) describes the single IV curve associated with values for parameters: I_L, I_O, R_s, R_{sh}, and n. To obtain a model for the electrical performance of a module over all irradiance and temperature conditions, Eq. (1) is supplemented with equations that define how each of the five parameters change with effective irradiance E (i.e., the irradiance that is converted to electrical current, which differs from plane-of-array irradiance due to reflection losses and spectral mismatch), and/or cell temperature T_C. These equations introduce additional parameters to the model, and variation among these equations gives rise to different performance models (e.g., [2], [6]). Here, we demonstrate our techniques using the performance model described by De Soto et al. [2] which supplements Eq. (1) with the following additional equations which involve an additional parameter E_{g0}:

$$I_L = I_L\left(E, T_C\right) = \frac{E}{E_0}\left[I_{L0} + \alpha_{Isc}\left(T_C - T_0\right)\right] \tag{2}$$

$$I_O = I_{O0}\left[\frac{T_C}{T_0}\right]^3 \exp\left[\frac{1}{k}\left(\frac{E_g\left(T_0\right)}{T_0} - \frac{E_g\left(T_C\right)}{T_C}\right)\right] \tag{3}$$

$$E_g\left(T\right) = E_{g0}\left(1 - 0.0002677\left(T_C - T_0\right)\right) \tag{4}$$

$$R_{sh} = R_{sh}\left(E\right) = R_{sh0}\left(E_0/E\right) \tag{5}$$

$$R_s = R_{s0} \tag{6}$$

$$n = n_0 \tag{7}$$

In Eq. (2) through Eq. (6)(7), the subscript $\sim_0$ indicates a value at the reference conditions of irradiance E_0 (1000 W/m^2) and cell temperature T_0 (298K); these values, i.e., n_0, I_{O0}, I_{L0}, R_{sh0}, R_{s0} and E_{g0} must be determined from a set of IV curves measured at various levels of irradiance and cell temperature. Other choices are available for these additional equations, the use of which results in different single-diode models (e.g., [6]; [7]).

III. Review of Parameter Estimation Methods

The literature describing proposed methods for extracting values for the five parameters appearing in Eq. (1) is extensive; as early as 1986, proposed methods were sufficiently numerous to merit comparative studies (e.g., [11]). Here, we do not attempt a comprehensive literature survey; instead we cite examples that illustrate different approaches to parameter estimation that we considered. We emphasize that all published methods we reviewed were successful in extracting parameters for which the computed IV curves reasonably matched the data. We considered these numerically successful methods in light of our objective: to outline a method by which the full performance model in [2] can be parameterized.

Some proposed methods (e.g., [12]; [13]) simplify or replace the diode equation Eq. (1)) to overcome its implicit nature before extracting parameters. We did not pursue these techniques, because fundamentally, they estimate parameter values for a model different than Eq. (1). We also do not consider methods that are formulated to use only information found on a typical manufacturer's data sheet, (e.g., [14]) although that problem is of significant practical interest.

Methods that consider the diode equation (Eq. (1)) and make use of measured IV curves fundamentally involve solving a system of non-linear equations by numeric methods. Typically, a system of non-linear equations is formulated by evaluating Eq. (1) at specific conditions to obtain equations corresponding to different points on the IV curve. For example, [2], [7] and [15] evaluate Eq. (1) at STC for the short-circuit, open-circuit and maximum power points, to obtain three equations involving five unknowns; a fourth equation is obtained by setting $dP/dV = 0$ (where $P = IV$) at the maximum power point, and a fifth equation is obtained by translating an IV curve to a cell temperature different from STC (using temperature coefficients determined by some other method). Other proposed methods obtain a system of equations by making approximations to Eq. (1) over parts of its domain (e.g., [16], [17]) or to equations derived from Eq. (1) (e.g., [18], [19], [20]). The system of equations is then solved by a numerical technique, such as root-finding (e.g., [4], [7]) or global optimization (e.g., [21], [22]) both of which involve iteration to (i) solve Eq. (1) for current (or voltage), (e.g., [4]) and (ii) adjust parameter values to minimize an error metric.

A challenge common to all techniques arises from the widely disparate magnitudes of terms appearing in Eq. (1). For V near V_{OC}, for a 72-cell module the argument $\left(V + IR_s\right)/N_S n V_{th}$ of the exponential term takes values on the order of 30 (i.e., $30 \approx \left(50 + IR_s\right)/\left(72 \times 1.1 \times 0.02\right)$). Unless R_{sh} is unreasonably small (i.e., on the order of 5Ω) so that the $\left(V + IR_s\right)/R_{sh}$ becomes comparable to the $I_L \approx 8\text{A}$, I_L must be offset by $I_O\left[\exp\left[\left(V + IR_s\right)/N_S n V_{th}\right] - 1\right]$ in order for current I to be near 0. Consequently in this region of the IV curve $I_O \approx \exp\left(-30\right) \approx 10^{-13}$, and relatively small changes in the estimated value for the diode factor n (e.g., from 1.1 to 1.15) cause large changes to the value for I_O (e.g., by a factor of more than 3). Multivariable optimization techniques that rely on derivatives (e.g., Newton's method) or on domain partitioning (e.g., Nelder-Mead method) may be challenged to update individual parameter values if not formulated appropriately.

When formulating the system of equations it is common to adopt the approximation (e.g., [19], [23]) $R_{sh} \approx -dV/dI$ at $I = I_{SC}$. From analysis of synthetic IV curves we found that using this approximation results in erroneous values for R_{sh} by as much as 20%. Error in one parameter induces errors of varying magnitudes in all other parameters, because parameter values are related via Eq. (1). The source of the error derives from the assumption that all terms other than R_{sh} in the exact expression ([10], Eq. 7) for the derivative are negligible, when in fact these terms may amount to a substantial fraction of R_{sh}.

Some parameter estimation methods (e.g., [16], [18], [24]) divide $\left[0, V_{OC}\right]$ into several intervals and formulate different systems of equations for each interval, within which the equations comprising each system (which may be simplified by approximations similar to those already discussed) result in better estimates of certain parameters (because data are confined to regions where those parameters are most influential). These methods are attractive because they are motivated by an understanding of the behavior of the physical system being modeled. However, they are difficult to formulate to be reproducible; the boundaries between the intervals comprising $\left[0, V_{OC}\right]$ are often determined by visual examination, and different choices of boundaries will result in different subsets of data being used to estimate each parameter with consequent differences in parameter values.

Among the surveyed literature we found several approaches ([25], [26], [27]) that consider the full range of each IV curve and make no simplifying approximations. A common attractive feature of these methods is their use of integrated data. Rather than estimating coefficients by fitting the diode equation (Eq. (1)) to data directly, [25] and [26] fit the integrated IV curve to corresponding integrated data via multiple linear regression; the two approaches differ in the variable of integration (voltage in the case of [25]; current in the case of [26]). Fitting to integrated data offers the advantage of suppressing the effects of random measurement error. In contrast, [27] estimates parameters by fitting the derivative dI/dV to measured data. Error in the measurements of current and/or voltage may be amplified by numerical differentiation; consequently, the method in [27] fits polynomials to the IV data before differentiation to smooth the effects of measurement error, a step which is not necessary with integral methods. We first implemented and tested the method in [25]. As discussed below, we found that the regression must be performed quite carefully, and even when this is done, too often the resulting parameter values were not physically meaningful (e.g., negative resistances).

IV. Parameter Estimation Method

We propose a sequential approach to obtaining parameter values from measured IV curves. Here, we briefly outline the parameter estimation process; a detailed description will be provided in a forthcoming report [28]. Throughout the process, we solve Eq. (1) using Lambert's *W* function (e.g., [10], Eq. 2 and Eq. 3) for which highly accurate numerical methods are available ([29], [30]) for a wide range of its argument, thus avoiding the need for an iterative solution of Eq. (1).

Step 1: We determine temperature coefficients from IV curves with irradiance near STC (i.e., 1000 W/m^2), using linear regression, as is commonly done ([5]; [31]). For the performance model outlined in Sect. II, only the temperature coefficient α_{Isc} is required, although Step 2 of our method also requires the temperature coefficient β_{Voc} for V_{OC}.

Step 2: We obtain the diode factor n from the relationship between V_{OC} and effective irradiance E. IV curves are required over a range of irradiance, preferably from 400 W/m^2 to 1000 W/m^2. The equation for V_{OC} in the Sandia Array Performance Model (SAPM) [32] is asymptotically the same as is obtained from Eq. (1) [28]. Thus, we use this equation to write

$$V_{OC} - \beta_{Voc}\left(T_C - T_0\right) = V_{OC0} + nN_S V_{th} \ln\left(E/E_0\right) \qquad (8)$$

where N_S is the number of cells in series and V_{th} is the thermal voltage at T_C per cell, then obtain n from the slope of a linear regression. Thus, the value for n is constant for all IV curves as expected by the performance model.

Step 3: For each IV curve, we sequentially determine values for R_{sh}, I_O, R_s and I_L:

Step 3a: R_{sh} is obtained using an approach modified from the integration method in [25]. In [25], the co-content integral is shown to be exactly equal to a polynomial in V and $I = I\left(V\right)$:

$$\begin{aligned} CC\left(V\right) = \int_0^V \left(I_{SC} - I\left(v\right)\right) dv = c_1 V + c_2\left(I_{SC} - I\right) \\ + c_3 V\left(I_{SC} - I\right) + c_4 V^2 + c_5\left(I_{SC} - I\right)^2 \end{aligned} \qquad (9)$$

As presented in [25], the integral in Eq. (9) is evaluated numerically, the coefficients c_i are determined by multiple linear regression, and values for all five parameters I_L, I_O, R_S, R_{sh}, and n are determined from the coefficients c_i. For example,

$$R_{sh} = 1/2c_4 \quad \text{([25], Eq. 11).} \qquad (10)$$

When applied to various sets of IV curves, we found the approach given in [25] to be problematic for R_{sh} and unreliable for the other parameter values. Investigation revealed that these problems resulted from numerical error in computing the integral for CC and from co-linearity between predictors in Eq. (9). For example, we found that simple trapezoid integration led to frequent failure to obtain reasonable parameter values and to systematic biases in the parameter values that were found. Numerical error in the integral was essentially overcome by first applying a spline interpolation method in [33]. Co-linearity effects were greatly reduced (but not eliminated) by a principal components transformation. With these improvements we generally obtained reasonable values for R_{sh} because this parameter depends on only one coefficient in Eq. (9). However, values for other parameters were much less reliable due to their dependence on several coefficients in Eq. (9).

Step 3b: I_O is initially estimated as:

$$I_O \approx \left(I_{SC} - V_{OC}/R_{sh}\right)\exp\left(-V_{OC}/nN_S V_{th}\right), \qquad (11)$$

which is obtained by evaluating Eq. (1) at V_{OC} and approximating $I_L + I_O \approx I_{SC}$. The value for I_O is then updated to minimize error in predicted V_{OC} using a root-finding technique akin to Newton's method.

Step 3c: R_S is initially estimated using the observed slope of the IV curve near V_{OC}. The slope $S(V)$ is computed at each voltage point V using a 5th order finite difference approach that does not require equally spaced voltages [34]. Then for a range of voltage $L < V < R$, a value $\hat{R}_S(V)$ is computed as

$$\hat{R}_S(V) = \frac{nN_S V_{th}}{I_{SC}} \ln\left[-\frac{nN_S V_{th}\left(R_{sh}S(V)+1\right)}{R_{sh}\hat{I}_O}\right] - \frac{V}{nN_S V_{th}} \qquad (12)$$

R_S is set equal to the average of $\hat{R}_S(V)$ where $\hat{R}_S(V)$ is positive, and is then updated to minimize error in predicted P_{MP} in a manner similar to the updating of I_O. We set $L = 0.5V_{OC}$ and $R = 0.9V_{OC}$, where the right limit is set to exclude points where the computed values of $S(V)$ become inaccurate. Care must be taken to exclude voltage points where the term $R_{sh}S(V)+1 > 0$ due to either a positive value for $S(V)$, indicative of questionable IV curve data, or a negative but very small value for $S(V)$, which may occur for V substantially less than V_{MP}. However, we also found it necessary that $L < V < R$ include voltages less than V_{MP}.

Step 3c: We determine I_L by evaluating Eq. (1) at short circuit conditions:

$$I_L = I_{SC} - I_O + \frac{I_{SC}R_s}{R_{sh}} + I_O \exp\left(\frac{I_{SC}R_s}{nN_S V_{th}}\right) \qquad (13)$$

Step 4: With values for I_L, I_O, R_s, R_{sh}, and n in hand for each IV curve, the remaining parameters in Eq. (2) through Eq. (7) are readily determined, using regression where needed. I_{O0} and E_{g0} are determined jointly by substituting Eq. (4) into Eq. (3), applying the natural logarithm, and performing a linear regression between $\ln I_O - 3\ln\left(T_C/T_0\right)$ and $\frac{1}{k}\left(\frac{1}{T_0} - \frac{1-0.000267\left(T_C - T_0\right)}{T_C}\right)$.

V. Parameter Estimation Demonstration

We tested our parameter estimation method using three data sets:

- Synthetic IV curves calculated for a wide range of parameter values;
- A set of 101 IV curves measured with a temperature-controlled flash solar simulator [31];
- A set of 4488 IV curves for the same module measured outdoors at Sandia's Photovoltaic System Evaluation Laboratory.

Testing with the set of synthetic IV curves confirmed that the method successfully recovered parameter values for IV curves with a wide range of characteristics, including: low and high series and shunt resistances; low and high fill factors; as well as high voltage, low current and low voltage, high current combinations.

The set of 101 IV curves were measured for a SunPower 305W crystalline silicon module ($V_{OC} = 65.0V$, $I_{SC} = 5.97A$ at STC) on a HALM flash solar simulator over a range of temperature and irradiance conditions generally consistent with the requirements of IEC 61853-1 [5], namely, irradiance varying from 200 W/m^2 to 1100 W/m^2 and module temperature varying from 25C to 75C. The set of 4488 IV curves were measured during March, 2012 in Albuquerque, NM. Parameter values extracted from both data sets are listed in Table 1. STC values agree within 1%. We found it necessary to regard E_{g0} as a fitting parameter rather than to use the value $E_{g0} = 1.121\text{eV}$ provided in [2]; otherwise, model fits to data were poor.

Table 1. Parameters estimated from indoor and outdoor data.

	Indoor	Outdoor		Indoor	Outdoor
I_{L0}	6.005A	6.017A	I_{SC0}	5.97A	5.97A
I_{O0}	0.139nA	0.356nA	V_{OC0}	64.8V	65.0V
n_0	1.074	1.12	I_{MP0}	5.63A	5.64A
R_{sh0}	1.28kΩ	1.50kΩ	V_{MP0}	53.7V	53.9V
R_{s0}	0.568Ω	0.521Ω	P_{MP0}	302.3W	303.7W
E_{g0}	0.994eV	0.956eV			

The estimated parameters were used with the model outlined in Sect. II to predict IV curves for the conditions observed during measurement of each IV curve. Fig. 1 and Fig. 2 display errors in predicted I_{MP}, V_{MP}, P_{MP}, and V_{OC} for indoor and outdoor data, respectively. Table 2 lists statistics for prediction errors. Errors quantified by mean bias (MBE) and root mean square deviation (RMSD) are small for all predicted quantities. Predicted V_{OC} and P_{MP} are generally unbiased, due to the optimization of I_O and R_S values to match these measured quantities. Some bias is present in the predicted I_{MP} and V_{MP} quantities as shown by the systematic trends in the errors for outdoor predictions (Fig. 2). It is not clear whether these biases result from the parameter estimation method, from systematic measurement error, or from a deficiency in the performance model itself.

Table 2. Statistics for prediction errors.

	Indoor data		Outdoor data	
	MBE	RMSD	MBE	RMSD
I_{MP}	8mA	16mA	34mA	27mA
V_{MP}	13mV	275mV	-220mV	185mV
P_{MP}	0.05W	1.0W	0.42W	1.4W
V_{OC}	9mV	49mV	4mV	120mV
I_{SC}	4mA	11mA	0.07mA	0.27mA

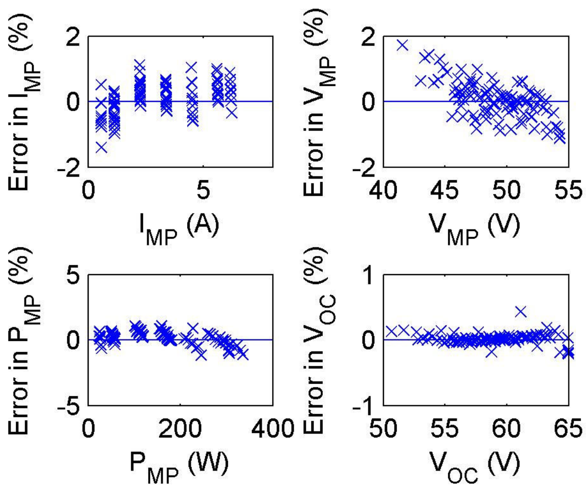

Fig. 1. Error in predicted voltage, current and power for model calibrated to indoor data.

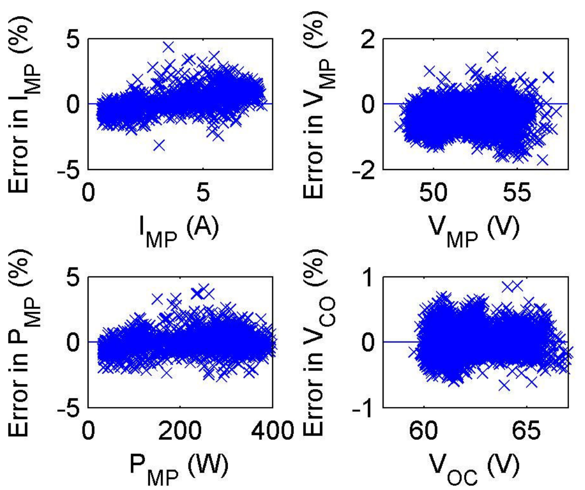

Fig. 2. Error in predicted voltage, current and power for model calibrated to outdoor data.

VI. Conclusion

We have demonstrated a parameter estimation method for a single diode module performance model that relies on, and uses, data across the range of each IV curve in a set of curves measured at a wide range of irradiance and temperature conditions. Using these data we calibrate the performance model to successfully predict performance at STC and at other conditions. Good agreement is observed between model predictions calibrated to indoor or outdoor data.

Acknowledgement

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

References

[1] A. Mermoud and T. Lejeune, "Performance Assessment Of A Simulation Model For Pv Modules Of Any Available Technology," presented at the 25th European Photovoltaic Solar Energy Conference, Valencia, Spain, 2010.

[2] W. De Soto, S. A. Klein, and W. A. Beckman, "Improvement and validation of a model for photovoltaic array performance," *Solar Energy,* vol. 80, pp. 78-88, 2006.

[3] J. L. Gray, "The Physics of the Solar Cell," in *Handbook of Photovoltaic Science and Engineering*, A. Luque, Hegedus, S., Ed., 2nd ed: John Wiley and Sons, 2011.

[4] A. P. Dobos, "An Improved Coefficient Calculator for the California Energy Commission 6 Parameter Photovoltaic Module Model," *Journal of Solar Energy Engineering,* vol. 134, 2012.

[5] I. E. C. (IEC), "61853-1 Ed. 1.0: Photovoltaic (PV) module performance testing and energy rating - Part I: Irradiance and temperature performance measurements and power rating," 2011.

[6] (2010). *User's Guide PVsyst Contextual Help*. Available: http://files.pvsyst.com/pvsyst5.pdf

[7] H. Tian, F. Mancilla-David, K. Ellis, E. Muljadi, and P. Jenkins, "A cell-to-module-to-array detailed model for photovoltaic panels," *Solar Energy,* vol. 86, pp. 2695-2706, 2012.

[8] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, "On the Lambert W Function " in *Advances in Computational Mathematics*, ed, 1996, pp. 329-359.

[9] A. Ortiz-Conde, F. J. García Sánchez, and J. Muci, "Exact analytical solutions of the forward non-ideal diode equation with series and shunt parasitic

resistances," *Solid-State Electronics,* vol. 44, pp. 1861-1864, 2000.
[10] A. Jain and A. Kapoor, "Exact analytical solutions of the parameters of real solar cells using Lambert W-function," *Solar Energy Materials and Solar Cells,* vol. 81, pp. 269-277, 2004.
[11] D. S. H. Chan and J. C. H. Phang, "Analytical methods for the extraction of solar-cell single- and double-diode model parameters from I-V characteristics," *Electron Devices, IEEE Transactions on,* vol. 34, pp. 286-293, 1987.
[12] S. Karmalkar and S. Haneefa, "A Physically Based Explicit Model of a Solar Cell for Simple Design Calculations," *Electron Device Letters, IEEE,* vol. 29, pp. 449-451, 2008.
[13] F. Bonanno, G. Capizzi, G. Graditi, C. Napoli, and G. M. Tina, "A radial basis function neural network based approach for the electrical characteristics estimation of a photovoltaic module," *Applied Energy,* vol. 97, pp. 956-961, 2012.
[14] G. Farivar and B. Asaei, "Photovoltaic module single diode model parameters extraction based on manufacturer datasheet parameters," in *Power and Energy (PECon), 2010 IEEE International Conference on*, 2010, pp. 929-934.
[15] J. A. Duffie and W. A. Beckman, *Solar Engineering of Thermal Processes* Wiley, 2006.
[16] W. Kim and W. Choi, "A novel parameter extraction method for the one-diode solar cell model," *Solar Energy,* vol. 84, pp. 1008-1019, 2010.
[17] J. C. H. Phang, D. S. H. Chan, and J. R. Phillips, "Accurate analytical method for the extraction of solar cell model parameters," *Electronics Letters,* vol. 20, pp. 406-408, 1984.
[18] E. Dallago, D. Finarelli, and P. Merhej, "Method based on single variable to evaluate all parameters of solar cells," *Electronics Letters,* vol. 46, pp. 1022-1024, 2010.
[19] K. I. Ishibashi, Y. Kimura, and M. Niwano, "An extensively valid and stable method for derivation of all parameters of a solar cell from a single current-voltage characteristic," *Journal of Applied Physics,* vol. 103, May 2008.
[20] S. K. Datta, K. Mukhopadhyay, S. Bandopadhyay, and H. Saha, "An improved technique for the determination of solar cell parameters," *Solid-State Electronics,* vol. 35, pp. 1667-1673, 1992.
[21] K. M. El-Naggar, M. R. AlRashidi, M. F. AlHajri, and A. K. Al-Othman, "Simulated Annealing algorithm for photovoltaic parameters identification," *Solar Energy,* vol. 86, pp. 266-274, 2012.
[22] E. Q. B. Macabebe, C. J. Sheppard, and E. E. van Dyk, "Parameter extraction from I–V characteristics of PV devices," *Solar Energy,* vol. 85, pp. 12-18, 2011.
[23] M. Chegaar, G. Azzouzi, and P. Mialhe, "Simple parameter extraction method for illuminated solar cells," *Solid-State Electronics,* vol. 50, pp. 1234-1237, 2006.
[24] F. Caracciolo, E. Dallago, D. G. Finarelli, A. Liberale, and P. Merhej, "Single-Variable Optimization Method for Evaluating Solar Cell and Solar Module Parameters," *Photovoltaics, IEEE Journal of,* vol. 2, pp. 173-180, 2012.
[25] A. Ortiz-Conde, F. J. García Sánchez, and J. Muci, "New method to extract the model parameters of solar cells from the explicit analytic solutions of their illuminated I–V characteristics," *Solar Energy Materials and Solar Cells,* vol. 90, pp. 352-361, 2006.
[26] L. Peng, Y. Sun, Z. Meng, Y. Wang, and Y. Xu, "A new method for determining the characteristics of solar cells," *Journal of Power Sources,* vol. 227, pp. 131-136, 2013.
[27] Y. Chen, X. Wang, D. Li, R. Hong, and H. Shen, "Parameters extraction from commercial solar cells I–V characteristics and shunt analysis," *Applied Energy,* vol. 88, pp. 2239-2244, 2011.
[28] C. Hansen, "Parameter Estimation for Single Diode Models of Photovoltaic Modules," Sandia National Laboratories, Albuquerque, NM, Forthcoming.
[29] D. A. Barry, P. J. Culligan-Hensley, and S. J. Barry, "Real Values of the W-Function," *ACM Transactions on Mathematical Software,* vol. 21, pp. 161-171, Jun 1995.
[30] D. A. Barry, S. J. Barry, and P. J. Culligan-Hensley, "Algorithm-743 - Wapr - a Fortran Routine for Calculating Real Values of the W-Function," *ACM Transactions on Mathematical Software,* vol. 21, pp. 172-181, Jun 1995.
[31] C. Hansen, D. Riley, and M. Jaramillo, "Calibration of the Sandia Array Performance Model Using Indoor Measurements " in *IEEE Photovoltaic Specialists Conference*, Austin, TX, 2012.
[32] D. L. King, E. E. Boyson, and J. A. Kratochvil, "Photovoltaic Array Performance Model," Sandia National Laboratories, Albuquerque, NM SAND2004-3535, 2004.
[33] L. Schumaker, "On Shape Preserving Quadratic Spline Interpolation," *SIAM Journal on Numerical Analysis,* vol. 20, pp. 854-864, 1983.
[34] M. K. Bowen and R. Smith, "Derivative formulae and errors for non-uniformly spaced points," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science,* vol. 461, pp. 1975-1997, July 8, 2005 2005.