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Sun-Relative Pointing for Dual-Axis Solar Trackers Employing Azimuth and Elevation Rotations

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Abstract

Dual axis trackers employing azimuth and elevation rotations are common in the field of photovoltaic (PV) energy generation. Accurate sun-tracking algorithms are widely available. However, a steering algorithm has not been available to accurately point the tracker away from the sun such that a vector projection of the sun beam onto the tracker face falls along a desired path relative to the tracker face. We have developed an algorithm which produces the appropriate azimuth and elevation angles for a dual axis tracker when given the sun position, desired angle of incidence, and the desired projection of the sun beam onto the tracker face. Development of this algorithm was inspired by the need to accurately steer a tracker to desired sun-relative positions in order to better characterize the electro-optical properties of PV and CPV modules.

CONTENTS

FIGURES

NOMENCLATURE

AOI	Angle of incidence
CPV	Concentrating photovoltaic
DOE	Department of Energy
LCPV	Low concentration photovoltaic
PV	Photovoltaic
SNL	Sandia National Laboratories
θ_{SA}	Sun azimuth
θ_{SE}	Sun elevation
θ_{TA}	Tracker azimuth rotation
θ_{TE}	Tracker elevation rotation
α	Angle of incidence, AOI
β	Angle of incidence direction on tracker face
$\hat{x}$	Reference unit vector for topocentric coordinate system, points east
$\hat{y}$	Reference unit vector for topocentric coordinate system, points north
$\hat{z}$	Reference unit vector for topocentric coordinate system, points up (toward zenith)
S	Sun pointing vector expressed in topocentric coordinate system
x_m	Unit vector in tracker plane, orthogonal to y_m and z_m
y_m	Unit vector normal to tracker plane, orthogonal to x_m and z_m
z_m	Unit vector in tracker plane towards top of tracker, orthogonal to y_m and x_m

1. INTRODUCTION

Dual axis trackers are common in the photovoltaic (PV) and concentrating photovoltaic (CPV) industries as a method of pointing at the sun to maximize solar energy collection. They are also common in laboratories which test and characterize PV and CPV modules as a method of controlling the solar radiation incident upon the device under test. Many of the dual axis trackers in use are of the azimuth/elevation type.

Sandia National Laboratories uses a dual axis azimuth/elevation tracker when characterizing the electro-optical response of a module to changes in the solar angle of incidence (AOI), i.e., the angle between the sun vector and the module's normal vector. Short-circuit current is measured as the module is steered away from an orientation normal to the sun; the changes in short-circuit current over a range of AOI can then be related to the fraction of sunlight reflected away from the module rather than being captured by the module [1]. For CPV, it may be desirable to measure performance aspects other than short-circuit current (e.g., maximum power or current at maximum power).

Most flat-plate PV modules exhibit isotropic response to AOI, that is, their response is the same regardless of the orientation of the sun beam relative to their surface. Thus, AOI alone was sufficient to parameterize the electrical response of most flat-plate PV modules. However, in some modules, especially low concentration PV modules, performance depends on both the AOI and on the orientation of sun vector relative the module face. To characterize these anisotropic modules, we define one additional angle to describe sun orientation, and present an algorithm for pointing an azimuth/elevation tracker to a desired position described in terms of these two angles.

1.1. Definitions

Here, we define the terms and coordinate system used to describe sun position and tracker orientation. We generally use a topocentric coordinate system; that is, the frame of reference for celestial bodies for an observer on the Earth's surface. We also introduce a tracker-relative coordinate system which is used to define AOI.

1.1.1. Sun position

We use a topocentric coordinate system (Figure 1) for describing the position of the sun in the sky. The solar elevation angle, θ_{SE}, is the angle between the observer's horizon and the sun, usually expressed in degrees, and is defined on the interval [-90°, 90°] (negative angles occur when the sun is below the observer's horizon). At sunrise and sunset, the solar elevation angle is 0°, and the solar elevation angle reaches a maximum at solar noon. The complement of the solar elevation angle is the solar zenith angle, θ_{SZ}, which is defined as the angle between the sun and a vector pointed directly overhead.

The solar azimuth angle, θ_{SA}, describes the direction of the sun as a bearing on the Earth's surface. As a bearing, the azimuth angle is defined as the number of degrees clockwise from true north and ranges over the interval [0°, 360°). When the sun is due north of the observer, the azimuth is 0°, when the sun is due east of the observer the azimuth is 90° (south = 180°, west = 270°).

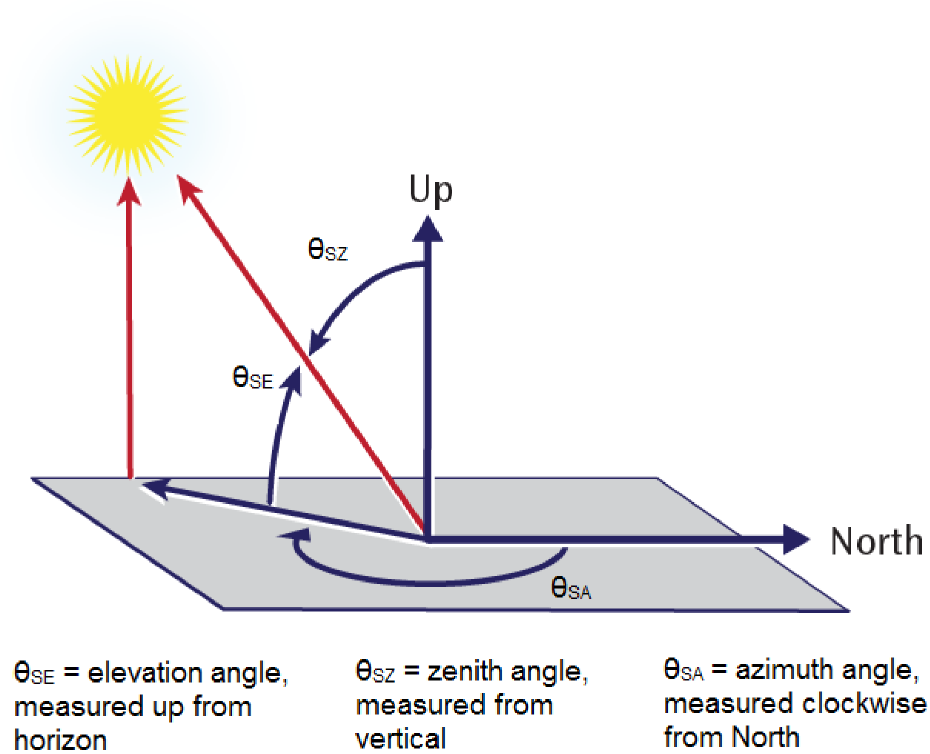

Figure 1: Description of solar position angles from an observer on Earth's surface

1.1.2. Tracker rotation

Azimuth/elevation trackers have two axes of rotation: one axis rotates the tracker around a vertical axis through all possible azimuth angles and the other axis rotates the tracker face about a horizontal axis through all possible elevation angles. The azimuthal rotation axis allows the tracker to point the tracker face through a range of azimuth angles, θ_{TA}, defined in the same manner as sun position: degrees clockwise from north. While the azimuth rotation of the tracker is defined over the interval [0°, 360°) most trackers, including those at SNL, are limited to a smaller range. For example, SNL's trackers are limited to azimuth angles between approximately 60° (east northeast) and 300° (west northwest).

A tracker's elevation rotation axis allows the tracker to point the tracker face through a range of elevation angles. The tracker's elevation angle, θ_{TE}, is defined as the angle between the horizontal vector in the direction of the tracker azimuth, and the vector normal to the tracker's face, with possible values in the interval (-180°, 180°]. This range, combined a possible range of [0°, 360°) range for tracker azimuth, means that the potential range of tracker pointing angles covers all possible pointing directions *twice*. However, because the tracker lacks a third rotation axis (around the normal to the plane of the tracker face), the tracker face is oriented differently for the two possible orientations which point to the same direction. For example, a tracker at pointing angles (θ_{TA}, θ_{TE}) = (100°, 0°) is pointing to the same celestial location as a tracker at pointing angles (280°, 180°), but the tracker face is inverted in the latter coordinates. Put another way, an arrow sketched onto the tracker face that points "up" (toward the sky) in the first set of pointing angles would point "down" (toward the ground) in the second set of pointing angles. As with azimuth angles, most azimuth/elevation trackers are mechanically limited in elevation angle to a range less than [-180°, 180°). The newest SNL research tracker (ATS 2) is limited to the elevation angle range [-10, 180]. Most trackers designed for solar energy collection are limited to elevation angle ranges approximately [5°, 90°] or less.

Many solar energy applications refer to the "tilt angle" or "slope" of a PV module or system from a horizontal plane. We note that the tilt angle of a tracker face and the tracker elevation angle are *not* the same, but are related through equation 1.

$$TiltAngle = |90° - \theta_{TE}| \quad (1)$$

1.1.3. Angle of incidence and angle of incidence direction

Solar angle of incidence (AOI), denoted here by α, is the angle between the module's normal vector and the vector pointing to the middle of the sun. We define α over the interval [0, 90) so that the beam of the sun is always striking the face of the module. PV modules which are mounted on dual axis trackers are typically mounted in the plane of the tracking face. For maximum solar energy collection, the tracker face is pointed toward the sun throughout the day and the normal vector of the PV module points at the sun. However, in research applications the PV module may be pointed away from the sun over a range of AOI to characterize the module's response to AOI.

As mentioned earlier, we have found that modules with anisotropic response to AOI cannot be adequately characterized with AOI alone. Therefore, we introduce the angle of incidence direction (AOI direction), β, defined by the projection onto the tracker face of the vector from the sun to the tracker. We quantify β over the interval [0, 360) in degrees counterclockwise from the line between the module's center and the module's "top" as illustrated in Figure 2. For example, consider the tracker pointing at an orientation (θ_{TA}, θ_{TE}) of (0°, 0°), i.e., the tracker face is a plane perpendicular to the earth's surface with the module's "top" being up and module normal pointed north. Consider a coordinate system on the module's face where 0° points "up" toward the zenith, 90° points east, 180° points down (toward Earth), and 270° points west. If a vector from the tracker face to the sun is projected (or "collapsed") onto the tracker face, β is the measure of the angle counterclockwise from 0° on the tracker face. Because the coordinate system is referenced to the tracker face, it moves relative to the earth as the tracker rotates.

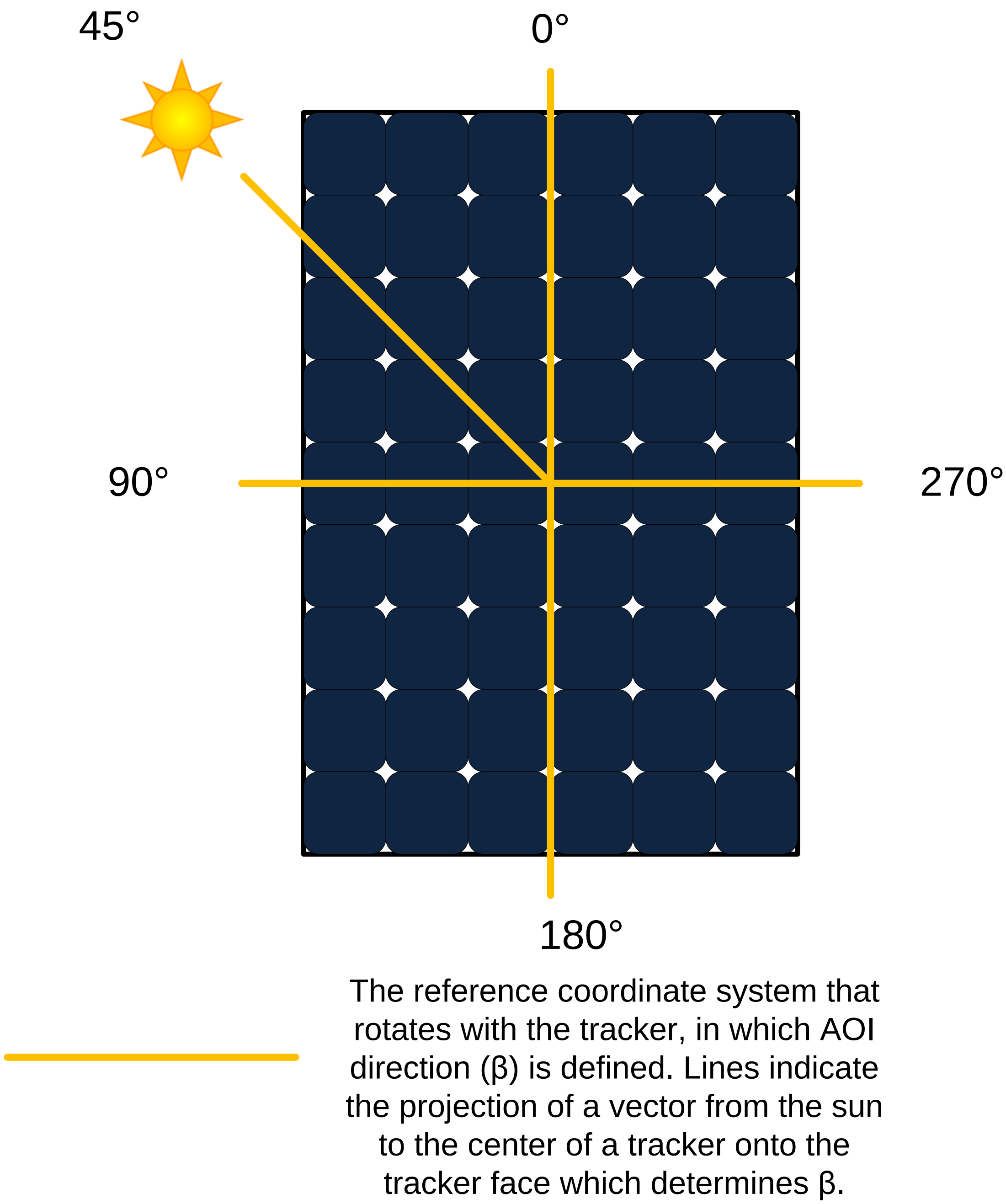

Figure 2: Description of the tracker-face coordinate system in which AOI direction is defined

2. CALCULATION OF VARIABLES

2.1. Calculating α and β given sun and tracker positions

Calculation of α and β from θ_{SE}, θ_{SA}, θ_{TA}, and θ_{TE} is relatively straightforward. In a topocentric, right-handed, Cartesian coordinate system with unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ where:

$$\hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \text{east} \tag{2}$$

$$\hat{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \text{north} \tag{3}$$

$$\hat{z} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \text{up} \tag{4}$$

The unit vector *S* pointing to the sun in the same topocentric coordinate system is:

$$S = \begin{bmatrix} \sin(\theta_{SA}) \times \cos(\theta_{SE}) \\ \cos(\theta_{SA}) \times \cos(\theta_{SE}) \\ \sin(\theta_{SE}) \end{bmatrix} \tag{5}$$

We also define a right-handed Cartesian coordinate system relative to the tracker face with unit vectors x_m, y_m, and z_m. The tracker face is a plane containing x_m and z_m, and the tracker normal vector is y_m. The vector z_m points from the tracker center to the tracker "top"; the vector x_m is 90° counterclockwise from z_m. Relative to the topocentric coordinate system, the vectors defining the tracker-relative coordinate system will rotate as the tracker rotates. After a tracker rotation in azimuth and/or elevation, the rotated tracker-relative unit vectors can be expressed in the topocentric coordinate system:

$$x'_m = \begin{bmatrix} \cos(\theta_{TA}) \\ -\sin(\theta_{TA}) \\ 0 \end{bmatrix} \tag{6}$$

$$y'_m = \begin{bmatrix} \sin\theta_{TA} \times \cos\theta_{TE} \\ \cos\theta_{TA} \times \cos\theta_{TE} \\ \sin\theta_{TE} \end{bmatrix} \tag{7}$$

$$z'_m = \begin{bmatrix} -\sin\theta_{TE} \times \sin\theta_{TA} \\ -\sin\theta_{TE} \times \cos\theta_{TA} \\ \cos\theta_{TE} \end{bmatrix} \tag{8}$$

As S is a unit vector pointing to the sun, and y'_m is a unit vector describing the rotated tracker normal in the same topocentric coordinates, the angle of incidence α may be found easily using equation 9.

$$\cos(\alpha) = S \cdot y'_m = \sin\theta_{SE} \times \sin\theta_{TE} + \cos\theta_{SE} \times \cos\theta_{TE} \times \cos(\theta_{SA} - \theta_{TA}) \tag{9}$$

where · is the usual dot product.

By projecting the sun vector S onto the rotated tracker surface, defined by x'_m and z'_m, we obtain the angle of incidence direction β as shown in equations 10 through 12.

$$(S \cdot x'_m) = \cos\theta_{SE} \times \cos\theta_{TA} \times \sin\theta_{SA} - \cos\theta_{SE} \times \cos\theta_{SA} \times \sin\theta_{TA} \tag{10}$$

$$(S \cdot z'_m) = \cos\theta_{TE} \times \sin\theta_{SE} - \sin\theta_{TE} \times \cos\theta_{SE} \times \cos(\theta_{SA} - \theta_{TA}) \tag{11}$$

$$\beta = \mathrm{atan2}[(S \cdot x'_m), (S \cdot z'_m)] \tag{12}$$

where atan2(y, x) is the four quadrant arctangent of $\frac{y}{x}$; for example, atan2(2, -3) ≈ 146.3°.

2.2. Calculating tracker position given sun position, α, and β

The inverse problem, calculating the appropriate tracker rotations θ_{TA} and θ_{TE} to achieve desired α and β, given θ_{SE} and θ_{SA}, is considerably more difficult and in fact may have more than one solution, or no solution. For example, if the sun is south at 45° elevation (i.e., $\theta_{SE} = 45°$, and $\theta_{SA} = 180°$) and it is desired that $\alpha = 45°$ and $\beta = 0°$ (i.e., *AOI* = 45° and AOI direction is towards the top of the tracker) then two solutions exist: $\theta_{TA} = 180°$ and $\theta_{TE} = 0°$ (i.e., the tracker normal is pointed south at the horizon and the tracker top is up), and $\theta_{TA} = 0°$ and $\theta_{TE} = 90°$ (i.e., the tracker normal is pointed straight upwards and the tracker top is pointed south). We calculate θ_{TA} and θ_{TE} using the following algorithm that accommodates cases where more than one solution, or no solution, exists. Because θ_{TA} and θ_{TE} are dependent upon sun position, the algorithm must be continually performed to accommodate the sun's movement through the sky (relative to the topocentric observer).

Generally, θ_{TA} and θ_{TE} may be found by simultaneously solving equations 9 and 12. In order to simplify the resulting expressions we make the following substitutions:

$$C = \cos\theta_{SE} \tag{13}$$

$$N = \sin\theta_{SE} \tag{14}$$

$$D = \tan\beta \tag{15}$$

$$A = \cos\alpha \tag{16}$$

$$U = \theta_{SA} - \theta_{TA} \quad (17)$$

and obtain from equations 9 and 12 two equations in the two unknowns θ_{TE} and U.

$$A = N \times \sin\theta_{TE} + C \times \cos\theta_{TE} \times \cos U \quad (18)$$

$$C \times \sin U = N \times D \times \cos\theta_{TE} - C \times D \times \sin\theta_{TE} \times \cos U \quad (19)$$

The solutions to equations 18 and 19 depend on the value of β.

2.2.1. General case when $tan\,\beta$ is defined

When the value for $D = \tan\beta$ is defined (i.e., $\beta \neq 90°$ and $\beta \neq 270°$), equations 18 and 19 admit a general solution for θ_{TE} and θ_{TA}. Solving equations 18 and 19 for θ_{TE} and U (using Maple™) then applying equation 17 yields the following:

$$\theta_{TE} = \operatorname{atan2}(F, R) \quad (20)$$

$$\theta_{TA} = \theta_{SA} - U = \theta_{SA} - \operatorname{atan2}(\mathrm{G}, \mathrm{H}) \quad (21)$$

where

$$F = \frac{D^2A^2+D^2N^2+A^2+N^2-R^2(D^2A^2+1)}{2AN(D^2+1)} \quad (22)$$

$$G = \frac{D^3N^2-D^3A^2-DA^2+DN^2+R^2(D^3A^2+D)}{(D^2+1)CNR} \quad (23)$$

$$H = \frac{A^2+D^2A^2-N^2-D^2N^2+R^2(D^2A^2+1)}{(D^2+1)CAR} \quad (24)$$

In equations 20 through 24, R is a root of the 4th order polynomial:

$$\mathrm{a}w^4 + \mathrm{b}w^2 + \mathrm{c} = 0 \quad (25)$$

where

$$\mathrm{a} = (D^2A^2 + 1)^2 \quad (26)$$

$$\mathrm{b} = -2(D^2 + 1)(A^4D^2 - A^2D^2N^2 - 2A^2N^2 + A^2 + S^2) \quad (27)$$

$$\mathrm{c} = (D^2 + 1)^2(A^2 - N^2)^2 \quad (28)$$

The polynomial in equation 25 admits 0, 2, or 4 real solutions (counting repeated roots). Because equation 25 is quadratic in form, we can classify the roots, and the solutions θ_{TE} and θ_{TA}, in terms of the discriminant, $\mathrm{b}^2 - 4\mathrm{ac}$.

Case 1: $\mathbf{b^2 - 4ac < 0}$

When $\mathrm{b}^2 - 4\mathrm{ac} < 0$, there are no real roots R and thus no possible solution for θ_{TE} and θ_{TA} given the sun position and the desired values of α and β.

Case 2: $\mathbf{b^2 - 4ac \geq 0}$

When $\mathrm{b}^2 - 4\mathrm{ac} \geq 0$ there are either two or four real roots R counting repeated values:

$$\left\{+\sqrt{\frac{-\mathrm{b}+\sqrt{\mathrm{b}^2-4\mathrm{ac}}}{2\mathrm{a}}}, -\sqrt{\frac{-\mathrm{b}+\sqrt{\mathrm{b}^2-4\mathrm{ac}}}{2\mathrm{a}}}, +\sqrt{\frac{-\mathrm{b}-\sqrt{\mathrm{b}^2-4\mathrm{ac}}}{2\mathrm{a}}}, -\sqrt{\frac{-\mathrm{b}-\sqrt{\mathrm{b}^2-4\mathrm{ac}}}{2\mathrm{a}}}\right\} \tag{29}$$

However, two of the possible solutions for θ_{TE} and $\theta_{TA}$are extraneous; we denote the values of R which correspond to the two actual solutions as

$$R_1 = \lambda_1\sqrt{\frac{-\mathrm{b}+\sqrt{\mathrm{b}^2-4\mathrm{ac}}}{2\mathrm{a}}} \tag{30}$$

$$R_2 = \lambda_2\sqrt{\frac{-\mathrm{b}-\sqrt{\mathrm{b}^2-4\mathrm{ac}}}{2\mathrm{a}}} \tag{31}$$

where λ_1 and λ_2 are either +1 or -1.

Values for λ_1 and λ_2 are found by:

$$\lambda_1 = \mathrm{sgn}(\cos\beta) \tag{32}$$

$$\lambda_2 = \mathrm{sgn}(\cos\beta) \times \mathrm{sgn}[-\cos(\alpha + \theta_{SE})] \tag{33}$$

where sgn(x) denotes the sign or signum function of x. Equations 30 through 33 were developed empirically by examining all four possible solutions deriving from equation 29, and selecting the two solutions which lie in the desired AOI direction (upwards on the module, corresponding to $\cos\beta \geq 0$, or downwards). The two extraneous solutions lie in the opposite directions.

When $\alpha + \theta_{SE} \neq 90°$ it can be shown that R_1 and R_2 are distinct, and hence there are two different solutions for $(\theta_{TE}, \theta_{TA})$ both of which satisfy equations 18 and 19. Thus, when $\alpha + \theta_{SE} \neq 90°$ and $\mathrm{b}^2 - 4\mathrm{ac} \geq 0$ there are two tracker pointing directions which provide the desired values for α and β; one obtained with $R = R_1$, and a second obtained with $R = R_2$.

When $\alpha + \theta_{SE} = 90°$, $\lambda_2 = 0$ from equation 33, thus $R_2 = 0$ also. For $R = R_2 = 0$ equations 23 and 24 are indeterminate and therefore provide no solution for θ_{TA}. In this case, one solution $(\theta_{TE}, \theta_{TA})$ results from $R = R_1$ in equations 20 through 24, and a second solution is trivial:

$$\theta_{TE} = 90° \tag{34}$$

$$\theta_{TA} = \beta - 180° + \theta_{SA} \tag{35}$$

2.2.2. Special case when $tan\,\beta$ is undefined

When $\beta = 90$ or $\beta = 270$, the value for $D = \tan\beta$ is undefined. In these cases, a separate calculation path is implemented in order to find θ_{TE} and θ_{TA} from a desired α and β, given θ_{SE} and θ_{SA}. When $\tan\beta$ is undefined, it must be that the sun vector, S, is perpendicular to the vector z'_m. From equation 9 and equation 11 we have the following system of equations in two unknowns, θ_{TA} and θ_{TE}:

$$0 = (S \cdot z'_m) = \cos\theta_{TE} \times \sin\theta_{SE} - \sin\theta_{TE} \times \cos\theta_{SE} \times \cos(\theta_{SA} - \theta_{TA}) \tag{36}$$

$$\cos\alpha = S \cdot y'_m = \sin\theta_{SE} \times \sin\theta_{TE} + \cos\theta_{SE} \times \cos\theta_{TE} \times \cos(\theta_{SA} - \theta_{TA}) \tag{37}$$

Eliminating $\cos(\theta_{SA} - \theta_{TA})$ from equations 36 and 37 obtains

$$\sin\theta_{TE} \times \cos\alpha = \sin\theta_{SE} \tag{38}$$

Solutions to equation 38 depend on the value of $\alpha + \theta_{SE}$.

2.2.2.1. $\boldsymbol{\beta = 90}$ or $\boldsymbol{\beta = 270}$ and $\boldsymbol{\alpha + \theta_{SE} > 90°}$

If $\alpha + \theta_{SE} > 90°$ then $0 > \cos(\alpha + \theta_{SE}) = \cos^2\alpha - \sin^2\theta_{SE}$; because both $0 < \alpha < 90°$ and $0 < \theta_{SE} \leq 90°$ it must be that $\sin\theta_{SE} > \cos\alpha$. Substituting into equation 38 we obtain

$$\sin\theta_{SE} = \sin\theta_{TE} \times \cos\alpha < \sin\theta_{TE} \times \sin\theta_{SE} \tag{39}$$

which leads to $\sin\theta_{TE} > 1$. Consequently, for cases where $\beta = 90$ or $\beta = 270$ and $\alpha + \theta_{SE} > 90°$, there are no solutions. That is, no values of θ_{TE} and θ_{TA} exist to give the desired α and β.

2.2.2.2. $\boldsymbol{\beta = 90}$ or $\boldsymbol{\beta = 270}$ and $\boldsymbol{\alpha + \theta_{SE} = 90°}$

If $\beta = 90$ or $\beta = 270$ and $\alpha + \theta_{SE} = 90°$, then $\cos\alpha = \cos(90° - \theta_{SE}) = \sin\theta_{SE}$, which in equation 38 implies $\sin\theta_{TE} = 1$. There exists only one solution as the tracker must be rotated to point at the zenith:

$$\theta_{TE} = 90° \tag{40}$$

$$\theta_{TA} = \beta - 180° + \theta_{SA} \tag{41}$$

2.2.2.3. $\boldsymbol{\beta = 90}$ or $\boldsymbol{\beta = 270}$ and $\boldsymbol{\alpha + \theta_{SE} < 90°}$

If $\beta = 90$ or $\beta = 270$ and $\alpha + \theta_{SE} < 90°$, there exist two solutions. The first solution allows the tracker to remain "upright", that is, $\cos\theta_{TE} \geq 0$, while the second forces the tracker to be "upside down". Both solutions for θ_{TE} follow from solving equation 38. Because $\alpha < 90° - \theta_{SE}$ and both $\theta_{SE} < 90°$ and $\alpha < 90°$, it follows that $\sin\theta_{SE} < \sin(90° - \alpha) = \cos\alpha$, thus equation 38 admits two solutions: one in Quadrant I or IV (equation 42)

$$\theta_{TE} = \sin^{-1}\left(\frac{\sin\theta_{TE}}{\cos\alpha}\right) = \sin^{-1}\left(\frac{N}{A}\right) \tag{42}$$

and a second solution in Quadrant II or III (equation 43):

$$\theta_{TE} = 180° - \sin^{-1}\left(\frac{N}{A}\right) \tag{43}$$

Note that equations 42 and 43 yield θ_{TE} values between -90° and 270°. Because we define θ_{TE} to values within the range (-180°, 180°], values in the range [180°, 270°] have 360° subtracted from them to be within the defined limits of θ_{TE}.

For each value of θ_{TE} found by equations 42 or 43, the corresponding value for θ_{TA} is then found from equation 44.

$$\theta_{TA} = \theta_{SA} - \cos^{-1}\left(\frac{A \times \cos\theta_{TE}}{C}\right) * \sin\beta \tag{44}$$

Equation 44 is obtained by eliminating terms involving θ_{TE} from the system comprising equations 36 and 37.

2.2.3. Special case when $tan\,\beta = 0$ and $\theta_{SE} = 90$

When the sun is directly overhead, that is $\theta_{SE} = 90$, it is clear that only values of β which may be accomplished by an azimuth/elevation tracker are $\beta = 0$ or $\beta = 180$. In the case of $\theta_{SE} = 90$ and $\beta = 0$ or $\beta = 180$, the discriminant for equation 25 will be equal to 0. The equations provided in section 2.2.1 will determine a correct value for θ_{TE}; however, the equations will also provide a value for θ_{TA}. Under these conditions the value provided for θ_{TA} is irrelevant, as rotations in the tracker azimuth do not result in changes of α or β.

3. PITFALLS WHEN COMPUTING VALUES NUMERICALLY

For applications of testing solar energy products, a typical implementation of these equations will require some form of computer. We have found several possible pitfalls to avoid when numerically evaluating for the solutions of the equations above. Most of these pitfalls arise due to precision errors when computers manipulate numbers that are then used in checks for equality or in inequalities.

3.1. Evaluating inverse cosine and inverse sine

Evaluations of inverse trigonometric functions can sometimes be problematic, especially when the arguments of the inverse function contain approximations of trigonometric functions, such as in equations 9, 42, and 43. It is possible, in some situations, to have arguments to the inverse cosine and inverse sine functions which are slightly above 1 or below -1, in which case the inverse trigonometric function could be improperly evaluated.

In these cases, it may be prudent to limit arguments to inverse sine and inverse cosine functions to the interval [-1, 1] prior to evaluation.

3.2. Comparison for equality

Some of the equations presented above involve evaluating for equalities or inequalities, for example, in section 2.2.2 there is a comparison to determine if $\alpha + \theta_{SE} = 90°$. In situations such as these, precision errors may again cause values which *should* be equivalent to evaluate as unequal. One possible solution to these precision errors is to evaluate if the values are nearly equivalent. For example, one may evaluate the equality comparison of $\alpha + \theta_{SE} = 90$ as $|\alpha + \theta_{SE} - 90| < \varepsilon$, where ε is a small positive number which is larger than the numeric precision of the calculation platform.

3.2. Evaluating when θ_{SE} is 0

As stated previously, the sun elevation, θ_{SE}, is defined over the interval [-90°, 90°], yet the equations listed above should only be used when $\theta_{SE} > 0$. If θ_{SE} is 0, then $\sin(\theta_{SE}) = N = 0$ and equations 22 and 23 are undefined. It is therefore recommended to set θ_{SE} to a small positive value (perhaps 0.0001) for extremely low sun angles; while doing so introduces some small error in the tracker pointing solution, the deviation is probably insignificant compared to error in the apparent sun position caused by miscalculation of atmospheric refraction or to effects introduced by near shading or far shading.

4. CONCLUSIONS

Sandia National Laboratories has developed a system whereby the angle between direct beam sunlight and a terrestrial plane (e.g. a photovoltaic module, solar tracker face) may be described both by the angle of incidence and the direction which the beam falls on the plane. These values, denoted α and β respectively, are simple to calculate using equations 9 through 12 when given the sun position and the pointing direction of a 2-axis solar tracker employing azimuth and elevation rotation axes.

It is more difficult, however, to determine the correct pointing angles, θ_{TA} and θ_{TE}, of an azimuth/elevation two-axis solar tracker that obtain a desired α and β for a given sun position. We describe an algorithm for determination of these tracker pointing angles when the sun is above the horizon. It is possible that there may be 0, 1, or 2 tracker orientations which provide the desired α and β, and the choice of which orientation is best for any application is left to the implementer.

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