

Frequency and Mode Shape Calculations for Vertical Axis Wind Turbines Using Reduction Techniques and Complex Eigensolvers

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Abstract

This study was done to assess the reliability of reduction techniques and complex eigensolvers when used to compute frequencies and mode shapes for finite element models of vertical axis wind turbines. Reduction schemes are a means to produce a lower order system of equations which accurately describe the dynamic characteristics of complex structures and hence reduce the cost of eigensolutions. Two reduction schemes - Guyan reduction and generalized dynamic reduction - and two eigensolvers - the inverse power method and the upper Hessenberg method - were examined. Although there are four possible combinations of reduction method and eigensolver, only two were extensively examined. These combinations were Guyan reduction plus the inverse power method and generalized dynamic reduction plus the upper Hessenberg method. Both combinations worked reliably for the calculation of frequencies and mode shapes of three different wind turbine models when certain basic guidelines were followed. The size of the reduced problem which can be obtained from a particular reduction scheme determines which computational approach is more efficient.

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INTRODUCTION

For Darrieus vertical axis wind turbines, the frequencies and mode shapes of the turbine vary as the rotational speed of the turbine changes. This phenomenon occurs because of rotating coordinate system effects [Reference 1]. It is important to accurately predict the frequencies and mode shapes corresponding to various rotational speeds in order to determine if the turbine will encounter any resonant conditions as it is brought up to operational speed and to make sure the desired operational speed is not close to any of the natural frequencies of the structure.

The mathematical description of a rotating wind turbine with a finite element formulation produces a problem that requires a complex eigensolver to determine mode shapes and frequencies [Reference 1]. A study has been conducted to determine the complex frequencies and mode shapes for several wind turbine models. In these studies, reduction techniques were used to reduce the size of the eigenvalue problem. Dynamic reduction techniques are used routinely in the aerospace industry. Aerospace structures are typically stiff structures, whereas vertical axis wind turbines are flexible structures. This study was conducted in large part to assess the reliability of using dynamic reduction techniques on such flexible structures. It was also conducted to determine if complex eigenvalue solution methods can reliably and consistently handle the eigenvalue problems generated from finite element models of vertical axis wind turbines.

DESCRIPTION OF COMPUTATIONAL PROBLEM AND SOLUTION METHODS

When a finite element model is generated for a vertical axis wind turbine including rotating coordinate system effects, the system of equations used to determine the frequencies and mode shapes is given by

$$[-\omega^2[M] + i\omega[C] + [K]]\{U\}e^{i\omega t} = 0.$$

The mass matrix is represented by $[M]$, and the stiffness matrix is represented by $[K]$. The $[K]$ matrix is the standard structural stiffness matrix except for the addition of terms which account for centrifugal effects. Both $[M]$ and $[K]$ are symmetric matrices. The Coriolis matrix, $[C]$, is skew symmetric if structural damping is ignored. The details for deriving this system of equations can be found in Reference [1]. The above problem yields complex frequencies and mode shapes. It can be transformed into a problem which yields real frequencies and mode shapes [Reference 2]. This latter method of calculating frequencies and mode shapes was not studied.

The structural analysis code MSC/NASTRAN was used to generate the matrices for the finite element model and to solve for the frequencies and mode shapes. MSC/NASTRAN, at the time of this report, has two dynamic reduction techniques - Guyan reduction and generalized dynamic reduction - and two complex eigensolution methods - the inverse power method and the upper Hessenberg method. The use of these techniques is described in the MSC/NASTRAN User's Manual [Reference 3] and the MSC/NASTRAN Application Manual [Reference 4].

Guyan reduction is an older technique than generalized dynamic reduction and has been more widely used. For Guyan reduction, the user selects the set of physical degrees of freedom to be used in the lower order model. Once this reduced set has been specified, a specific procedure involving matrix partitioning, multiplication, and addition is used to condense out all other degrees of freedom which are not in the reduced model. Guyan reduction has two main drawbacks. First, the user must be careful in the selection of the reduced set or poor estimates will be obtained for the frequencies and mode shapes. This is not too serious since there are some guidelines which will help the user select a reasonable reduced set for most structures. Second, Guyan reduction cannot be used, in general, to easily produce "small" reduced models, especially when compared to the reduced models which can be obtained by generalized dynamic reduction. Some idea of what constitutes small will come about in a discussion of the results obtained in this study. Generalized dynamic reduction overcomes the problems that the user experiences with Guyan reduction. In generalized dynamic reduction, the user specifies a range of frequencies for which the calculated values

for the natural frequencies within this range must be accurate. The code then sets about constructing the reduced model in a manner similar to subspace iteration. For most applications, the number of degrees of freedom in the reduced model (referred to as generalized coordinates) is 1.5 times the number of desired system frequencies. Generalized dynamic reduction tends to be a more expensive reduction scheme computationally than Guyan reduction. This is due largely to the fact that generalized dynamic reduction is an iterative scheme rather than a sequence of matrix operations which is carried out only once as in Guyan reduction.

The two complex eigensolvers in MSC/NASTRAN, like the dynamic reduction schemes, have differences between them. The inverse power method is an extended core solver, which means it can handle very large systems of equations. Although it can handle large systems of equations, it should be noted that carrying out the actual solution of such a system would be very expensive computationally. The inverse power method does require the user to estimate the regions in the complex plane where the eigenvalues are likely to occur. For wind turbine problems, this is not particularly difficult since the eigenvalues all lie along the imaginary axis in the complex plane for all nonzero frequencies. The inverse power method will compute only those eigenvalues and associated eigenvectors specified by the user. The upper Hessenberg method is, at the time of this report, an in-core solver. (It could be modified to operate in an extended core model.) This method will automatically give all the eigenvalues for a system and as many associated eigenvectors as the user requests.

Because of the characteristics of the reduction schemes and the eigensolvers, it was decided to select two combinations for the wind turbine studies. The inverse power method would be used on reduced systems obtained by Guyan reduction, and the upper Hessenberg method would be used on reduced models obtained by generalized dynamic reduction. For the same accuracy, Guyan reduction typically results in systems requiring the larger equation handling capabilities of the inverse power method. Because larger systems of equations are being handled by this combination of reduction and solution, the situation is such that the reduction is computationally inexpensive and the eigensolution is expensive. The combination of generalized dynamic

reduction and the upper Hessenberg method operates in the opposite mode in terms of where the most computational effort is required. For wind turbine problems, generalized dynamic reduction can easily produce a reduced model which can be handled in-core by the upper Hessenberg method. Because the reduced model is small, the eigensolution will be relatively inexpensive. This latter combination represents, therefore, an expensive reduction scheme and an inexpensive eigensolution. It turns out to be the most highly automated scheme of the two. The only requirement on the user is that he or she specify the range of frequencies of interest.

Ideally, one would like to be able to produce a very small reduced model with Guyan reduction that would permit the use an efficient in-core solver such as the upper Hessenberg method. Under these circumstances, both the reduction scheme and eigensolution would be relatively inexpensive. More comments will be made on this particular combination in the concluding remarks.

WIND TURBINE MODELS USED IN THE STUDY

Three different wind turbine models have been used in the study. One of the models has two blades and the other two models have three blades. All three of the models are examined for static conditions only (i.e., $[C] = 0$ and $[K]$ is the standard stiffness matrix). Line drawings of these three models are shown in Figures 1 through 3. Each turbine is supported at the top by guy cables. These cables are represented by springs in the finite element model and are not shown in the figures.

The wind turbine shown in Figure 3 has a large number of pinned joints. The pin axis is always normal to the plane of the blade. The struts extending from the blades to the tower are pinned at the tower-strut and blade-strut junctions. The blades are pinned at the tower-blade junctions and at the points where the struts attach into the blades. This particular model has redundant frequencies because of the three-bladed symmetry. The models shown in Figures 1 and 2 have moment resisting joints where the blades attach to the tower. The approximate number of nodes and degrees of freedom for each model are indicated in each of the figures.

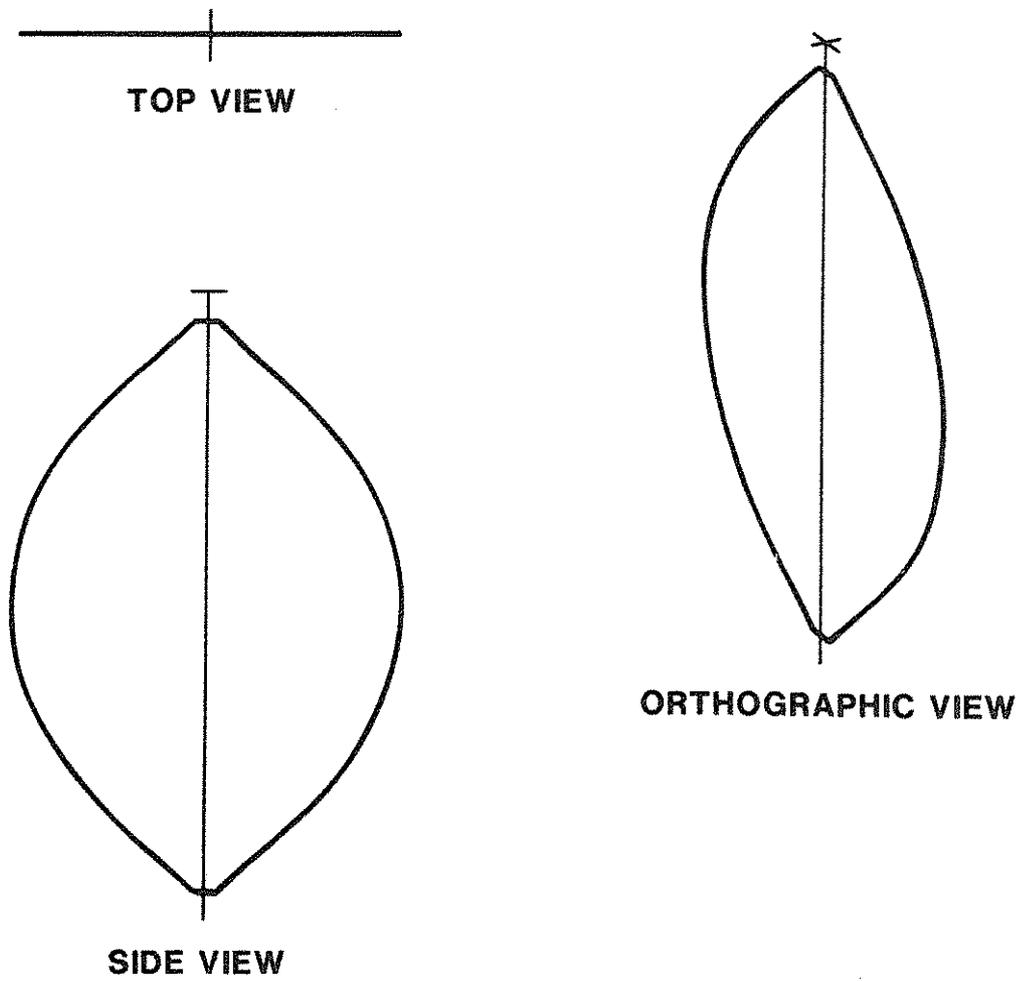


Figure 1. Two-Bladed Wind Turbine-Model 1. There are approximately 120 nodes (720 degrees of freedom) in the model.

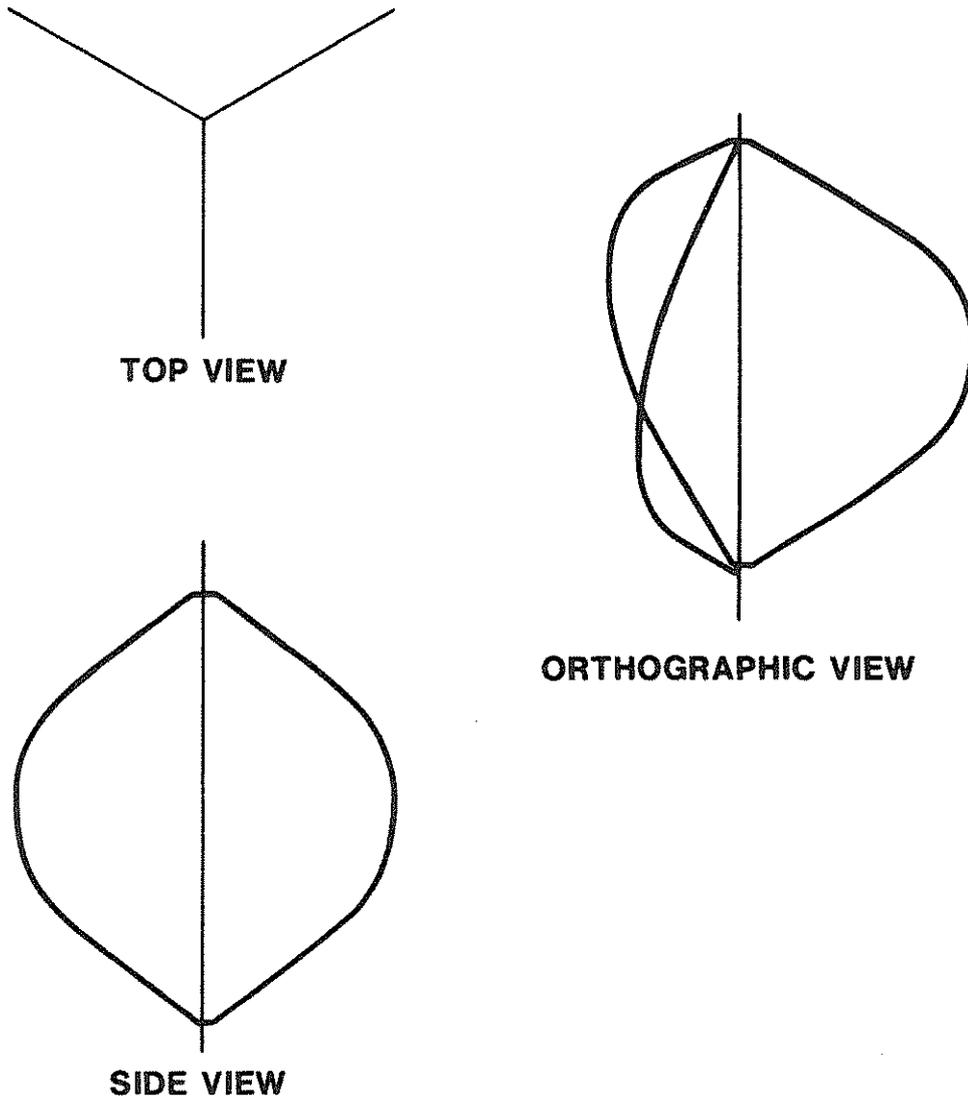
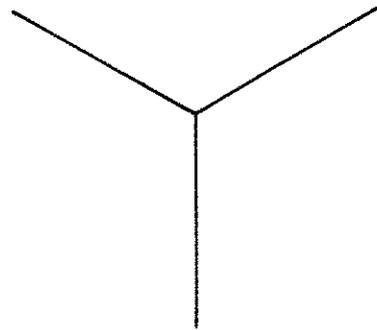
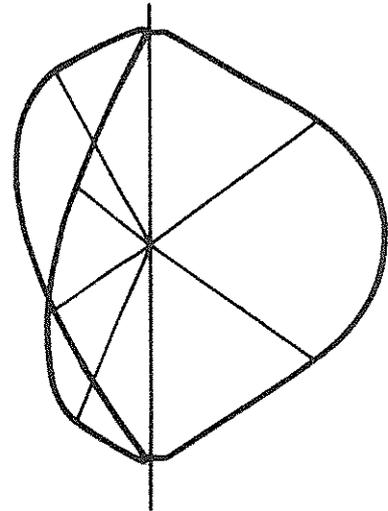


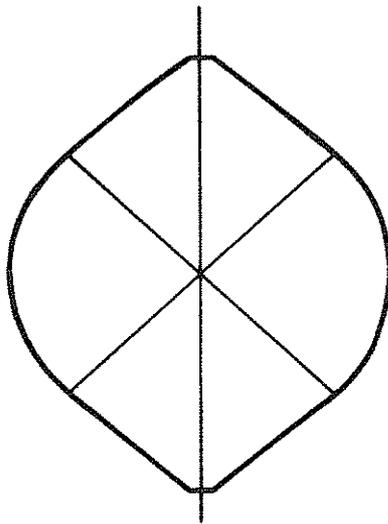
Figure 2. Three-Bladed Wind Turbine - Model 2. There are approximately 160 nodes (960 degrees of freedom) in the model.



TOP VIEW



ORTHOGRAPHIC VIEW



SIDE VIEW

Figure 3. Three-Bladed Wind Turbine-Model 3. There are approximately 180 nodes (1080 degrees of freedom) in the model.

Although only static conditions were considered, the complex eigensolvers in MSC/NASTRAN were still used to calculate the frequencies and mode shapes. The eigensolvers were being applied to a specialized case of the rotating wind turbine problem. The case of $[C] \neq 0$ will be considered in a future study.

RESULTS OF THE STUDY

Tables 1 through 3 show the calculated natural frequencies for the three different wind turbine models. Each table contains the eigenvalue results from a number of different reduced order models obtained from a number of computational approaches. The header of each column shows the dynamic reduction scheme employed, the number of degrees of freedom in the reduced model, and the eigensolver which was used. For the examples where Guyan reduction was used, the number of physical degrees of freedom selected to represent the reduced model are indicated. For those examples where generalized dynamic reduction was used, the header indicates the number of generalized coordinates as well as the number of physical degrees in the reduced problem; the number of degrees of freedom in the reduced model is the total of the physical degrees of freedom and the generalized coordinates.

The physical degrees of freedom were included when generalized dynamic reduction was used because of a phenomenon noticed early in the study of the two-bladed model. When generalized dynamic reduction was used without the inclusion of any physical degrees of freedom, certain mode shapes which were known to be symmetric about a plane passing through the tower and perpendicular to the plane of the blades were showing slight but noticeable asymmetries. The corresponding frequencies showed no errors and agreed quite well with the frequencies calculated by a variety of methods. An examination of the results showed that there were very slight asymmetries in the computed mode shapes which were being magnified in the plotting process by a scale factor. It was discovered that the plotted mode shapes could be "cleaned up" simply by including a few physical degrees of freedom in the system which conveyed some basic geometrical information

Table 1. Two-Bladed Wind Turbine - Model 1

Number of nodes \approx 120

Number of degrees of freedom in static model \approx 720

Computed Frequencies for Reduced Models

(a) denotes reduction technique

(b) denotes solution method for complex eigenvalue problem

(a) Guyan reduction 57 physical dof	(a) Guyan reduction 57 physical dof	(a) Guyan reduction 99 physical dof	(a) Generalized dynamic reduction 34 generalized coordinates 0 physical dof	(b) Inverse power	(b) Inverse power	(b) Upper Hessenberg	(b) Upper Hessenberg	Frequency (Hertz)	Frequency (Hertz)	Frequency (Hertz)	Frequency (Hertz)
1.64202	1.64202	1.64169	1.64161	1.64202	1.64169	1.64202	1.64161	1.64161	1.64169	1.64161	1.64161
1.77878	1.77876	1.77815	1.77771	1.77878	1.77815	1.77878	1.77771	1.77771	1.77815	1.77878	1.77771
1.79669	1.79675	1.79611	1.79568	1.79669	1.79611	1.79669	1.79568	1.79568	1.79611	1.79669	1.79568
2.64752	2.64763	2.64656	2.64620	2.64752	2.64656	2.64752	2.64620	2.64620	2.64656	2.64752	2.64656
3.63389	3.63390	3.62799	3.62454	3.63389	3.62799	3.63390	3.62454	3.62454	3.62799	3.63389	3.62799
3.74007	3.73990	3.73535	3.73217	3.74007	3.73535	3.73990	3.73217	3.73217	3.73535	3.74007	3.73535
3.99267	3.99274	3.99150	3.99083	3.99267	3.99150	3.99274	3.99083	3.99083	3.99150	3.99267	3.99150
	4.78798		4.78372			4.78798	4.78372	4.78372			4.78372
	5.06364		5.05563			5.06364	5.05563	5.05563			5.05563
	5.67818		5.65730			5.67818	5.65730	5.65730			5.65730
	5.94832		5.91064			5.94832	5.91064	5.91064			5.91064
	6.15418		6.13508			6.15418	6.13508	6.13508			6.13508
	7.08085		7.07358			7.08085	7.07358	7.07358			7.07358
	8.53267		8.43959			8.53267	8.43959	8.43959			8.43959
	8.56645		8.47205			8.56645	8.47205	8.47205			8.47205
	8.78460		8.72533			8.78460	8.72533	8.72533			8.72533
	11.74250		11.49049			11.74250	11.49049	11.49049			11.49049
	11.92076		11.62613			11.92076	11.62613	11.62613			11.62613
	12.54066		12.37654			12.54066	12.37654	12.37654			12.37654
	13.44309		13.29078			13.44309	13.29078	13.29078			13.29078
	15.14229		14.89642			15.14229	14.89642	14.89642			14.89642
	16.22764		15.27075			16.22764	15.27075	15.27075			15.27075
	16.46990		15.58949			16.46990	15.58949	15.58949			15.58949
	21.90958		19.91454			21.90958	19.91454	19.91454			19.91454
	22.44894		19.93273			22.44894	19.93273	19.93273			19.93273
	22.88053		21.37248			22.88053	21.37248	21.37248			21.37248
	23.14451		22.00702			23.14451	22.00702	22.00702			22.00702
	24.86066		24.63473			24.86066	24.63473	24.63473			24.63473

Table 1. (Continued) Two-Bladed Wind Turbine - Model 1

Number of nodes \approx 120
 Number of degrees of freedom in static model \approx 720

Computed Frequencies for Reduced Models

(a) denotes reduction technique
 (b) denotes solution method for complex eigenvalue problem

(a) Generalized dynamic reduction (a) Generalized dynamic reduction
 18 generalized coordinates 15 generalized coordinates
 0 physical dof 12 physical dof
 (b) Upper Hessenberg (b) Upper Hessenberg

Frequency (Hertz)
1.64216
1.77770
1.79590
2.64849
3.64174
3.74949
3.99323
4.78518
5.05657
5.65902
5.92147
6.13883
7.07477
8.49978

Frequency (Hertz)
1.64246
1.77931
1.79726
2.64880
3.63273
3.73905
3.99410
4.80867
5.05578
5.66534
5.92701
6.14991
7.13491
8.52029
8.56351
8.86884
11.52852
11.63075
12.74417

Frequency (Hertz)
1.64247
1.77932
1.79727
2.64884
3.64722
3.73926
3.99412
4.81484
5.05579
5.66760
5.92712
6.15269
7.14048
8.56540
8.82855
8.98075
11.54158
11.67597
12.80949

Table 2. Three-Bladed Wind Turbine - Model 2

Number of nodes \approx 160

Number of degrees of freedom in static model \approx 960

Computed Frequencies for Reduced Models

(a) denotes reduction technique

(b) denotes solution method for complex eigenvalue problem

(a) Guyan reduction 94 physical dof	(a) Guyan reduction 120 physical dof	(a) Generalized dynamic reduction 18 generalized coordinates
(b) Inverse power	(b) Inverse power	(b) Upper Hessenberg 15 physical dof

<u>Frequency (Hertz)</u>	<u>Frequency (Hertz)</u>	<u>Frequency (Hertz)</u>
1.97125	1.97079	1.97084
1.97126	1.97080	1.97134
1.97770	1.97723	1.97735
2.48715	2.48678	2.48721
2.48716	2.48693	2.48724
4.04315	4.04208	4.04434
4.04326	4.04220	4.04469
4.68326	4.66855	4.66733
4.71047	4.69705	4.69530
4.71059	4.69718	4.70389
6.17854	6.17478	6.17447
6.77062	6.76007	6.77912
6.77088	6.76035	6.78683
7.88581	7.79308	7.77506
7.88646	7.79579	7.77972
7.90455	7.79644	7.79915
8.33659	8.31876	8.34038
8.33713	8.31927	8.34310
12.51856	11.93957	11.98611
12.54435	11.96716	12.27006
12.54460	11.96733	12.97143
14.64507		14.90310

Table 3. Three-Bladed Wind Turbine - Model 3 (Hinged Joints Between Members)

Number of nodes \approx 180			
Number of degrees of freedom in static model \approx 1080			
Computed Frequencies for Reduced Models			
(a) denotes reduction technique	(a) Generalized dynamic reduction	(a) Generalized dynamic reduction	(a) Generalized dynamic reduction
(b) denotes solution method for complex eigenvalue problem	42 generalized coordinates	27 generalized coordinates	27 generalized coordinates
	15 physical dof	15 physical dof	15 physical dof
	(b) Upper Hessenberg	(b) Upper Hessenberg	(b) Upper Hessenberg
(a) Guyan reduction			
96 physical dof			
(b) Inverse power			
	<u>Frequency (Hertz)</u>	<u>Frequency (Hertz)</u>	<u>Frequency (Hertz)</u>
2.56189	2.56081	2.56093	2.56093
2.56208	2.56107	2.56113	2.56113
3.37122	3.34281	3.34281	3.34281
3.38182	3.35350	3.35351	3.35351
3.38207	3.35376	3.35376	3.35376
3.48576	3.45276	3.45276	3.45276
3.48631	3.45305	3.45305	3.45305
3.48645	3.45320	3.45320	3.45320
3.57143	3.51809	3.51809	3.51809
3.58276	3.52995	3.52995	3.52995
3.58563	3.53267	3.53268	3.53268
3.75738	3.69655	3.69665	3.69665
3.77172	3.71146	3.71146	3.71146
3.77180	3.71156	3.71156	3.71156
4.27105	4.27246	4.27281	4.27281
4.27568	4.27709	4.27753	4.27753
4.28131	4.28264	4.28316	4.28316
4.64949	4.64205	4.64262	4.64262
4.65789	4.65039	4.65078	4.65078
6.84762	6.79098	6.80842	6.80842
6.89716	6.79783	6.80876	6.80876
7.91949	6.80848	6.80965	6.80965
7.91956	7.83806	7.88852	7.88852
	7.86118	7.89276	7.89276
	10.67002	10.71886	10.71886
	10.98440	11.13682	11.13682
	11.05460	11.23173	11.23173
	11.22352	11.23304	11.23304
	13.99301	14.04153	14.04153
	14.00083	14.16034	14.16034

Table 3. (Continued) Three-Bladed Wind Turbine - Model 3 (Hinged Joints Between Members)

Number of nodes = 180
 Number of degrees of freedom in static model = 1080

Computed Frequencies for Reduced Models

- (a) denotes reduction technique
- (b) denotes solution method for complex eigenvalue problem

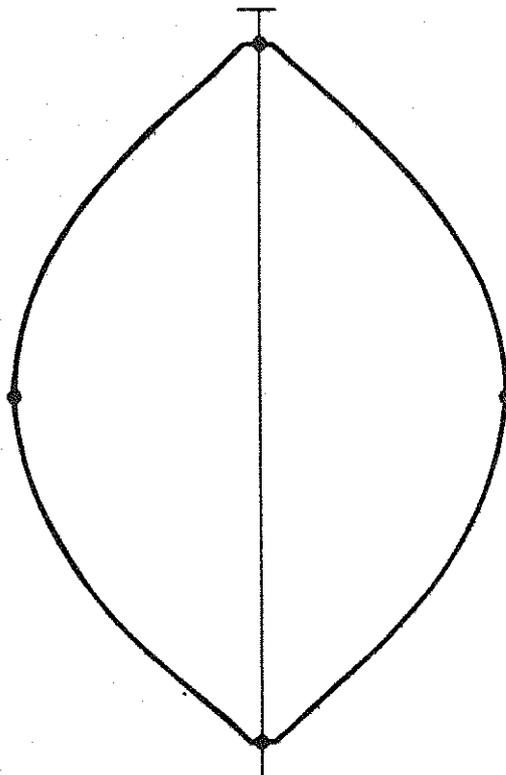
- (a) Generalized dynamic reduction
 - 9 generalized coordinates
 - 36 physical dof
- (b) Upper Hessenberg

Frequency (Hertz)
2.56653
2.56685
3.34284
3.35410
3.35434
3.45331
3.45437
3.45479
3.53499
3.56366
3.58049
3.73386
3.74541
3.76758
4.27269
4.27678
4.28200
4.71416
4.72178
7.05045
7.15760
7.52862
8.57294
8.67822
10.74897
11.13416
11.23178
11.23281
15.06500
15.81696
15.85994

about the wind turbine. For the two-bladed model, the translational degrees of freedom for node points at the top and bottom of the tower and the tips of the blades (see Figure 4) were included in the reduced model. This particular selection for the physical degrees of freedom reflects the basic symmetries in the structure. A similar approach was used on the three-bladed models. Translational degrees of freedom near the top and bottom of the tower and at the tips of the blades were included in the reduced models.

A number of general observations can be made about the results shown in the tables. First, for any one particular model, the various computational schemes produced calculated values for the model which agree quite well with one another. Second, the reduced models are considerably smaller than the full models set up for static analysis. The full models range from 720 to 1080 degrees of freedom; the reduced models range from 25 to 120 degrees of freedom. The smallest reduced model, 25 degrees of freedom, was obtained with generalized dynamic reduction. The reduced models from generalized dynamic reduction ranged from approximately one third to one half the size of the reduced models obtained from Guyan reduction.

The three bladed model with the large number of pinned conditions was the most sensitive of the three problems which were studied. The first reduced set selected for the Guyan reduction for this model did not yield all of the frequencies which were known to exist in the frequency range of interest. The specification for the reduced set was then changed slightly (the number of degrees of freedom in the set remained constant) and all of the desired frequencies were then obtained in the range of interest. The number of degrees of freedom for the reduced set for both of these cases (100 degrees of freedom) was at the lower limit of what can be reasonably used for the model. Consequently, even though the problem showed a sensitivity to the selection of the reduced set, it was for a reduced size specification that was lower than what one would normally use. In the application of generalized dynamic reduction to this model, the first reduced model included a high proportion of physical degrees of freedom to generalized coordinates. Physical degrees of freedom were specified not only at the tower and blade locations, but also on the struts. This also



**• INDICATES POSITIONS OF GRID POINTS ON MODEL WHICH
HAVE TRANSLATIONAL DEGREES OF FREEDOM INCLUDED
IN REDUCED MODEL**

Figure 4. Locations of Grid Points on Model 1 where Physical Degrees of Freedom were Specified for Inclusion in the Reduced Model to be Generated by Generalized Dynamic Reduction.

generated a condition similar to the first attempt with Guyan reduction - not all of the known frequencies in the desired range were obtained. This situation was corrected by removing the physical degrees of freedom associated with the struts. This particular example is shown in the last column of Table 3. Generalized dynamic reduction seems to work best when the number of physical degrees of freedom in the reduced model is held to a minimum.

CONCLUSIONS

Guyan reduction and generalized dynamic reduction are both capable of generating reduced order models for dynamic analysis from large models of vertical axis wind turbines which will produce accurate values for frequencies and mode shapes. One should observe the caveats and suggestions as outlined in the MSC/NASTRAN User's Manual and MSC/NASTRAN Applications Manual. Other than this, however, one does not encounter any special or troublesome problems when applying these reduction techniques to vertical axis wind turbine models.

For a production environment (i.e., an environment where people are examining different designs or several iterations of one design and computing frequencies and mode shapes for many rotational speeds), the combination of generalized dynamic reduction and the upper Hessenberg method is a very useful scheme since it is highly automated. It requires only the input of the frequency range of interest and the specification of a very few physical degrees of freedom. The combination of Guyan reduction and the inverse power method requires not only the input of the frequency range of interest, but also the specification of the reduced model by the user.

Guyan reduction can be used to produce a reliable reduced model which is small enough to be handled by in-core solvers if the reduced set is chosen carefully by an experienced analyst. The analyst must give careful consideration to the selection of the final reduced set and should have a basic understanding of the behavior of the wind turbine under consideration. If a large number of runs are to be made for one design and Guyan reduction can produce a reliable reduced model which can be handled by an in-core solver, then the use of Guyan reduction with the in-core solver will result

in quick and inexpensive computer runs for calculation of mode shapes and frequencies.

The analyst should pay attention to the size of the reduced model which can be obtained for a particular wind turbine. The time required to calculate mode shapes and frequencies rises sharply as the size of the reduced set increases. If a much smaller reduced set can be obtained with generalized dynamic reduction as opposed to Guyan reduction, then the most efficient overall approach will most likely involve the use of generalized dynamic reduction. Generalized dynamic reduction is the more expensive reduction method, but, if it leads to a significantly smaller set of equations to be handled by the eigensolver, the total cost for reduction and eigensolution may be smaller than a scheme which utilizes on Guyan reduction.

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