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Fourier Coefficients of Aerodynamic Torque Functions for the DOE/ Sandia 17-M Vertical Axis Wind Turbine

Gerald M. McNerney

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FOURIER COEFFICIENTS OF AERODYNAMIC TORQUE FUNCTIONS FOR THE DOE/SANDIA 17-M VERTICAL AXIS WIND TURBINE*

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ABSTRACT

The spectral characteristics of the aerodynamic torque on wind turbines are important in assessing drivetrain performance. This paper describes a Fast Fourier Transform method to deduce Fourier coefficients for the periodic torque functions predicted by aerodynamic theories for Darrieus-type rotors. The method is applied to show spectral characteristics of the torque on the DOE/Sandia 17-m Darrieus rotor predicted by the single and multiple streamtube aerodynamic models.

*The work described in this paper was performed for Sandia Laboratories under Contract No. 13-2322.

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Introduction

The 17-m vertical-axis wind turbine (VAWT) is a Darrieus-type wind turbine with a height-to-diameter ratio of 1, and troposkien* airfoil blades attached to a rotating vertical shaft (Figure 1). Aerodynamic forces acting on the blades produce torque on the center shaft, a torque that then passes through a speed increaser to rotate a high-speed shaft. This high-speed shaft in turn drives an ac induction motor/generator or a synchronous generator to produce power. The main or low-speed shaft for this size turbine will generally rotate at speeds varying from 30 to 55 rpm, while the generator maintains the high-speed or generator shaft at a constant rotational speed. The generator operates near the synchronous speed of 1800 rpm, controlled by the frequency of the utility line. Power is generated when the generator works to keep the rotational speed from exceeding its operating rpm, while power is consumed if the generator must work to keep the rpm from going below its operational rpm.

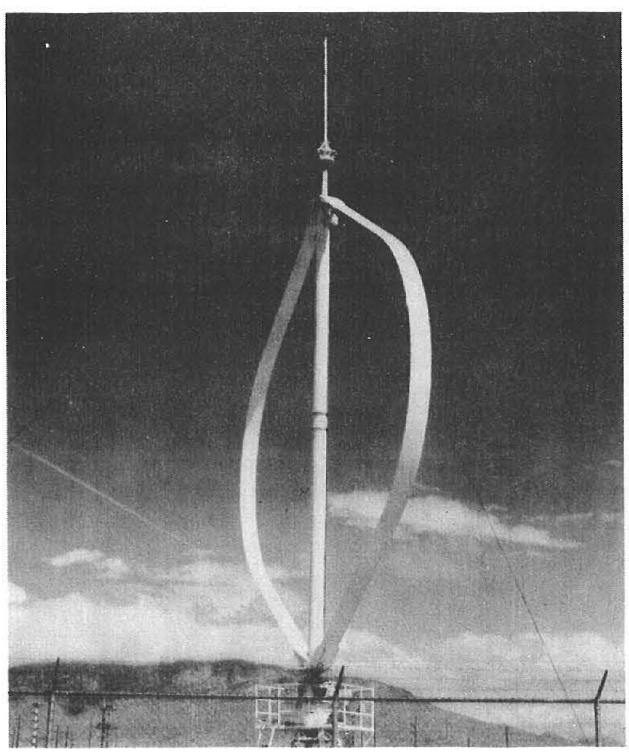


Figure 1. The 17-m VAWT

*The curved shape of a skipping rope

The aerodynamic torque on the blades produced by wind varies because the angle of attack of the wind on the turbine changes as the blades rotate as well as because of fluctuations in windspeed. Since we are concerned here only with the former effect, we will assume that the wind remains constant over a rotational cycle of the turbine.

On the 17-m VAWT, a torque meter is located on the main shaft on a rotating part below the point where the blades meet the shaft (Figure 2). This torque meter is equipped with a device to send an analog signal to a control room, where a minicomputer reads the signal through an analog-to-digital converter. Time series of torque as well as windspeed are created and stored on disk files through this system.

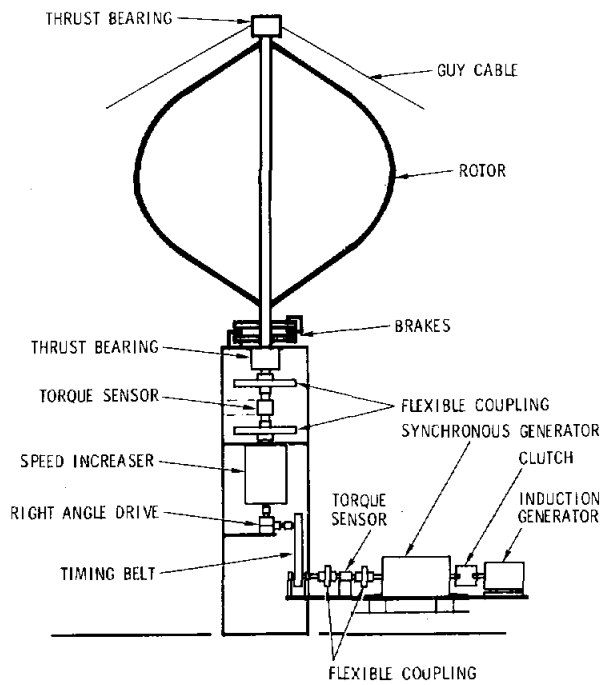


Figure 2. Schematic of 17-m VAWT and Drive Train

The fluctuation of aerodynamic torque caused by the changing angle of attack is called torque ripple. Torque ripple is important in considering not only the quality of power produced but also the fatigue life of various components along the drivetrain.

Aerodynamic models are available that predict the torque produced on the blades' position relative to wind direction. In most common use are streamtube-type models.[1] These models predict, for two-bladed rotors, an aerodynamic torque function which is periodic about 180-deg, and symmetric about 90-deg of rotor rotation. The symmetry results from the assumption in the streamtube models that the induced velocity is the same through the upstream and downstream faces of swept area. As Figure 3 illustrates, since the blade is unskewed and symmetric, and everywhere tangent to rotor swept area, the tangent force experienced at position θ by an element of the blade will be the same as the tangent force experienced by the blade element at position $-\theta$*.

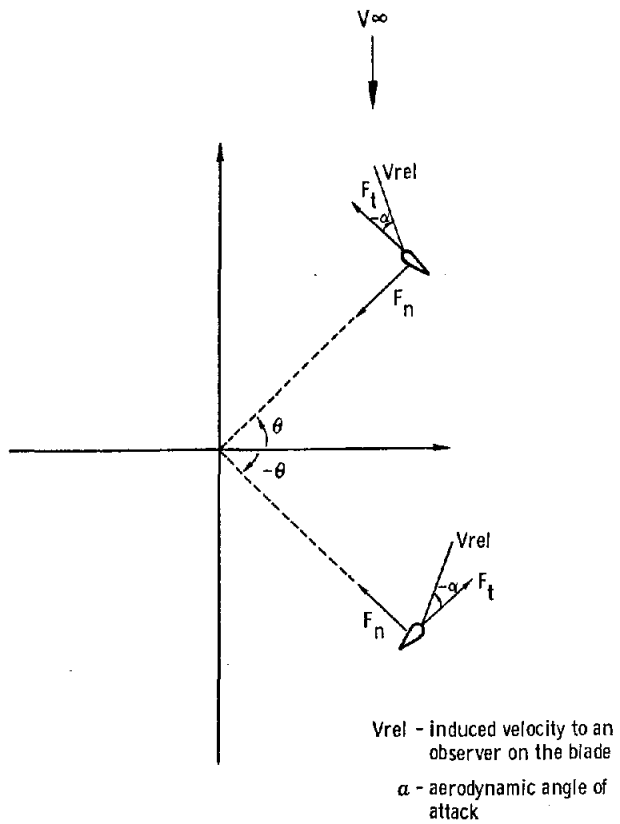


Figure 3. Symmetry of Streamtube-Type Aerodynamic Models

Because of this symmetry, the torque function may be expressed as a Fourier cosine series:

$$TQ(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{P} t\right) \quad , \tag{1}$$

where P = torque period.

Mechanical models may be devised which, when applied to the harmonic components of aerodynamic torque, yield the harmonic components of torque read at the drive train torque sensor.[1] That is, application of the drivetrain models will give

$$A_o = A_o(a_o) \, , \quad A_1 = A(a_1) \, , \quad A_2 = A_2(a_2) \quad ,$$

so that the torque at the torque meter will be

$$TQM(t) = \frac{A_o}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2\pi n}{P} t\right) \quad . \tag{2}$$

---

*The vortex-type aerodynamic models now under development do not assume the induced velocity is uniform throughout the swept area, and therefore for vortex models $TQ(\theta) \neq TQ(-\theta)$.

Unfortunately, aerodynamic torque obtained from the aerodynamic models is in the form of a time series and the Fourier series were not available. Consequently, early results used the first order approximation*

$$TQ(t) = \frac{a_0}{2} + a_1 \cos\left(\frac{2\pi}{P} t\right)$$

in estimating the torque ripple at the torque meter. Therefore, fine comparisons of predicted vs measured values of torque ripple were not available until a method for determining the complete Fourier cosine series had been developed.

The next section explains how Fourier trigonometric series coefficients were determined from the Fourier Transform and describes uses for the Fast Fourier Transform (FFT). The third section presents some results of the method applied to aerodynamic torque time series and compares single and multiple streamtube models.[2] The final section discusses the results and the method in general.

Analysis

Continuous Case

Any aerodynamic torque function is sufficiently well-behaved (e.g., differentiable and absolutely integrable in a finite interval) so that any analytical operations such as forming the Fourier Inverse, or interchanging the order of summation and integration of associated Fourier series, may be freely carried out.

Our primary objective is to find the coefficients a_0, a_1, a_2, ..., b_1, b_2, b_3, ... of the Fourier trigonometric series representation of the aerodynamic torque function, TQ(t). It is well known that, under the conditions stated above, if TQ(t) is periodic, with period P, then the series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos\left(\frac{2\pi n}{P} t\right) + b_n \sin\left(\frac{2\pi n}{P} t\right) \right]$$

converges to TQ(t), where the constants a_n and b_n are defined by

$$a_n = \frac{2}{P} \int_{-0}^{P} TQ(t) \cos\left(\frac{2\pi n}{P} t\right) dt \tag{3}$$

*As will be shown below, this approximation is quite valid for Darrieus-type rotors operating at tip speed ratios above the aerodynamic stall point.

and

$$b_n = \frac{2}{P} \int_{-0}^{P} TQ(t) \sin\left(\frac{2\pi n}{P} t\right) dt \ . \tag{4}$$

In view of the Euler relation

$$e^{i\theta} = \cos\theta + i \sin\theta$$

the trigonometric series representation of TQ(t) may be written in complex form

$$TQ(t) = \sum_{-\infty}^{\infty} \alpha_n \, e^{\frac{2\pi n}{P} it} \tag{5}$$

where

$$\alpha_n = \frac{1}{2}(a_n - ib_n) = \frac{1}{P} \int_0^P TQ(t) \, e^{-\frac{2\pi n}{P} it} \ . \tag{6}$$

If TQ(t) is an even function of t; that is, if

$$TQ(-t) = TQ(t) \ ,$$

then Eq (5) reduces to the cosine series,

$$TQ(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{P} t\right) .$$

When the functional form of TQ(t) is known, Eqs (3) and (4) may be used to determine the constants a_n, b_n. If the functional form of TQ(t) is not known, but TQ(t) is given (for example, as a discrete time series), it may be impractical to use Eq (1) to determine the Fourier coefficients, since a numerical integration scheme requires a large number of data points in a fundamental period of TQ(t) to achieve reasonable accuracy.

If the Fourier Transform of TQ(t) can be found, the constants are determinable because the Fourier Transform representation of a function reduces to the Fourier trigonometric series when the function is periodic. To see this, let f(t) be P periodic, and consider the Fourier Transform representation of f(t):

$$f(t) = \int_{-\infty}^{\infty} e^{2\pi i \lambda t} F(\lambda) \, d\lambda \tag{7}$$

where

$$F(\lambda) = \int_{-\infty}^{\infty} e^{-2\pi i\lambda t} f(t)\, dt \,. \tag{8}$$

Since f(t) is periodic, it admits the representation of Eq (5):

$$f(t) = \sum_{-\infty}^{\infty} \alpha_n e^{\frac{2\pi n}{P} it}$$

Substituting this into Eq (8), we obtain

$$F(\lambda) = \int_{-\infty}^{\infty} e^{-2\pi i\lambda t} f(t)\, dt$$

$$= \int_{-\infty}^{\infty} e^{-2\pi i\lambda t} \sum_{-\infty}^{\infty} \alpha_n e^{\frac{2\pi n}{P} it}\, dt$$

$$= \sum_{-\infty}^{\infty} \alpha_n \int_{-\infty}^{\infty} e^{2\pi it\left(\frac{n}{P} - \lambda\right)} dt$$

$$= \sum_{-\infty}^{\infty} \alpha_n \delta\left(\frac{n}{P} - \lambda\right) ,$$

where $\delta\left(\frac{n}{P} - \lambda\right)$ is the so-called delta function that satisfies

$$\delta\left(\frac{n}{P} - \lambda\right) = 0 \quad \text{for } \lambda \neq \frac{n}{P} ,$$

and (9)

$$\int_I \delta\left(\frac{n}{P} - \lambda\right) d\lambda = 1 \quad \text{for } \frac{n}{P} \epsilon I \,.$$

For more information on the delta function, see References 3 and 4.

Thus,

$$F(\lambda) = \sum_{-\infty}^{\infty} \alpha_n \delta\left(\frac{n}{P} - \lambda\right) , \tag{10}$$

and substituting this into Eq (7), we obtain

$$f(t) = \int_{-\infty}^{\infty} \sum_{-\infty}^{\infty} \alpha_n \delta\left(\frac{n}{P} - \lambda\right) e^{2\pi i \lambda t} \, d\lambda$$

$$= \sum_{-\infty}^{\infty} \alpha_n e^{\frac{2\pi n}{P} i t} ,$$

where Eq (9) has been used.

Now, $F(\lambda)$ is defined by Eq (8) as the Fourier Transform of f(t), but from Eq (10) we see that the particular form of $F(\lambda)$ when f(t) is periodic explicitly involves the complex Fourier coefficients α_n.

The next case to be considered, in which the function under consideration, TQ(t), is discrete, involves a different analysis, but the result is the same. The Fourier coefficients α_n, may be found by computing the discrete Fourier Transform of TQ(t).

Discrete Case

Let TQ(t) be a discrete function defined by

$$TQ(t) = \sum_{k=1}^{N} f_k \chi_{I_k}(t) \tag{11}$$

where

$$I_k = [t_{k-1}, t_k], \quad t_k = \frac{kT}{N} = k\Delta t ,$$

and $\chi_{I_k}(t)$ is the indicator function

$$\chi_{I_k}(t) \begin{cases} 0 \text{ if } t \notin I_k \\ 1 \text{ if } t \in I_k \end{cases} .$$

If we let $tq(\lambda)$ be the Fourier Transform of TQ(t), then by definition

$$tq(\lambda) = \int_{-\infty}^{\infty} TQ(t) \, e^{-2\pi i \lambda t} \, dt$$

$$= \int_{-\infty}^{\infty} \sum_{k=1}^{N} f_k \chi_{I_k}(t) \, e^{-2\pi i \lambda t} \, dt$$

$$= \sum_{k=1}^{N} f_k \int_{t_{k-1}}^{t_k} e^{-2\pi i\lambda t}\, dt = \sum_{k=1}^{N} \frac{f_k\, e^{\frac{-2\pi i\lambda kT}{N}}}{-2\pi i\lambda} \left(1 - e^{\frac{2\pi i\lambda T}{N}}\right)$$

$$= \frac{\left(1 - e^{\frac{2\pi i\lambda T}{N}}\right)}{-2\pi i\lambda} \sum_{k=1}^{N} f_k\, e^{\frac{-2\pi i\lambda kT}{N}} . \tag{12}$$

Now, suppose TQ(t) is periodic over M increments; that is, $TQ(t_{k+M}) = TQ(t_k)$, or $f_{k+M} = f_k$, and in addition that $\frac{N}{M} = J$, an integer. Then $tq(\lambda)$ may be simplified as follows:

$$tq(\lambda) = \frac{\left(1 - e^{\frac{2\pi i\lambda T}{N}}\right)}{-2\pi i\lambda} \left[\sum_{k=1}^{M} f_k\, e^{\frac{-2\pi i\lambda kT}{N}} + \sum_{k=1}^{M} f_k\, e^{\frac{-2\pi i\lambda (M + k)T}{N}} + \ldots + \sum_{k=1}^{M} f_k\, e^{\frac{-2\pi i\lambda (M(J - 1) + k)T}{N}}\right]$$

$$= \frac{\left(1 - e^{\frac{2\pi i\lambda T}{N}}\right)}{-2\pi i\lambda} \left[\sum_{k=1}^{M} f_k \sum_{L=0}^{J-1} e^{\frac{-2\pi i\lambda (LM + k)T}{N}}\right]$$

$$= \frac{\left(1 - e^{\frac{2\pi i\lambda T}{N}}\right)}{-2\pi i\lambda} \left(\sum_{L=0}^{J-1} e^{\frac{-2\pi i\lambda LMT}{N}}\right)\left(\sum_{k=1}^{M} f_k\, e^{\frac{-2\pi i\lambda kT}{N}}\right) \tag{13}$$

For a discrete function over the interval $T = N\Delta t$, the sum over L in Eq (13) may be determined exactly for the N frequencies $\lambda_r = \frac{r}{N\Delta t} = \frac{r}{T}$, $r = 1, 2, \ldots, N$. Two separate cases emerge, first when r is an integer multiple of J, $r = H \cdot J$, $H = 1, \ldots, M$, and, second, when r is not an integer multiple of J. In the first case,

$$\lambda_r = HJ/T, \text{ so } \frac{-2\pi i\lambda_r LMT}{N} = \frac{-2\pi i LH \cdot J \cdot M}{N} .$$

But

$$N = JM , \text{ so,}$$

$$\frac{-2\pi i\lambda_r LMT}{M} = -2\pi i LH ,$$

and

$$e^{\frac{-2\pi i\lambda_r LMT}{N}} = 1 ,$$

thus

$$\sum_{L=0}^{J-1} e^{\frac{-2\pi i\lambda_r LMT}{N}} = J \quad .$$

It follows that

$$tq(\lambda_r) = \frac{1 - e^{\frac{2\pi iH}{M}}}{-2\pi iH} T \sum_{k=1}^{M} f_k e^{\frac{-2\pi iHk}{M}} \quad . \tag{14}$$

On the other hand, when r is not an integer multiple of J, the sum over L in Eq (13) vanishes.

Proof: $$\frac{-2\pi i\lambda_r LMT}{N} = \frac{-2\pi iLr}{J}$$

So

$$\sum_{L=0}^{J-1} e^{\frac{-2\pi i\lambda\ TL}{J}} = \sum_{L=0}^{J-1} e^{\frac{-2\pi irL}{J}}$$

Now, let

$$\beta = \sum_{L=0}^{J-1} e^{\frac{-2\pi irL}{J}}$$

Then

$$e^{\frac{-2\pi ir}{J}} \beta = \sum_{L=1}^{J} e^{\frac{-2\pi irL}{J}} = \beta - 1 + e^{-2\pi i} \quad .$$

Solve for β,

$$\beta = \frac{1 - e^{-2\pi ir}}{1 - e^{\frac{-2\pi ir}{J}}} ,$$

where division is justified since r/J is not an integer. But $e^{-2\pi ir} = 1$, and thus $\beta = 0$. Since β is a factor of $tq(\lambda)$ in Eq (13), it follows that $tq(\lambda_r) = 0$. We therefore obtain the result that, in the case of a discrete function that is periodic and defined over an integral number of periods, the Fourier Transform is nonzero only for the N frequencies, λ_r, that are multiples of the fundamental frequency.

On the other hand, we may compute the coefficients of the Fourier series directly from the definition:

$$\alpha_n = \frac{1}{P} \int_0^P TQ(t)\, e^{\frac{-2\pi n i}{P} t}$$

$$= \frac{1}{P} \int_0^{T_0} \sum_{k=1}^{N} f_k \chi_{I_k} e^{\frac{-2\pi i n}{P} t}\, dt$$

$$= \frac{1}{P} \sum_{k=1}^{M} f_k \int_{t_{k-1}}^{t_k} e^{\frac{-2\pi i n}{P} t}\, dt = \sum_{k=1}^{M} - \frac{f_k}{2\pi i n} \left(e^{\frac{-2\pi i n}{P} t_k} - e^{\frac{-2\pi i n}{P} t_{k-1}} \right)$$

$$= \sum_{k=1}^{M} - \frac{f_k}{2\pi n i} e^{\frac{-2\pi i n k}{M}} \left(1 - e^{\frac{2\pi i n}{M}} \right)$$

$$= - \frac{\left(1 - e^{\frac{2\pi n i}{M}} \right)}{2\pi n i} \sum_{k=1}^{M} f_k\, e^{\frac{-2\pi i n k}{M}} . \tag{15}$$

where $P = M\Delta t$. Comparing this to Eq (14), we find

$$\alpha_n = \frac{tq(\lambda_r)}{T} ,$$

when

$$n = H \text{ and } r = H \cdot J \quad .$$

In actual practice, the FFT is used to compute the Fourier Transform of a discrete function. The FFT method was developed specifically for use with digital equipment. The FFT computational procedure involves some interchanging of matrix elements to reduce the number of calculations required for transforming a discrete function with N points from order N^2 to order N. In its simplest form, the FFT requires the transformed function to have 2^L points; that is, $N = 2^L$ for some integer L. Additionally, the FFT uses $\Delta t = T/N = 1$, which must be compensated for at the end of the calculation. If α_n is to be calculated using the FFT, Eq (16) can be used with N = T -- that is,

$$\alpha_n = \frac{tq(\lambda_r)}{N} = \frac{tq(\lambda_r)}{2^L} ,$$

where

$$\lambda_r = \frac{HJ}{N\Delta t} = \frac{HJ}{T} .$$

For more information on the FFT, see References 4, 5, or 6. A copy of the FFT FORTRAN IV subroutine used is given in Appendix A.

Results

The FFT method for determining Fourier coefficients has been applied to several particular time series of aerodynamic torque. The aerodynamics group at Sandia Laboratories calculated the time series by using the Multiple Streamtube Aerodynamic Model. This model uses conservation of momentum along with the airfoil lift and drag data in an iterative scheme to calculate the force at points along a turbine blade. Reference 7 describes the Multiple Streamtube Model, and Reference 8 describes the calculation particulars. The forces calculated for each point along the blade are then integrated over the blade length and the results added over the number of turbine blades to find the resultant torque at the center shaft as a function of time.

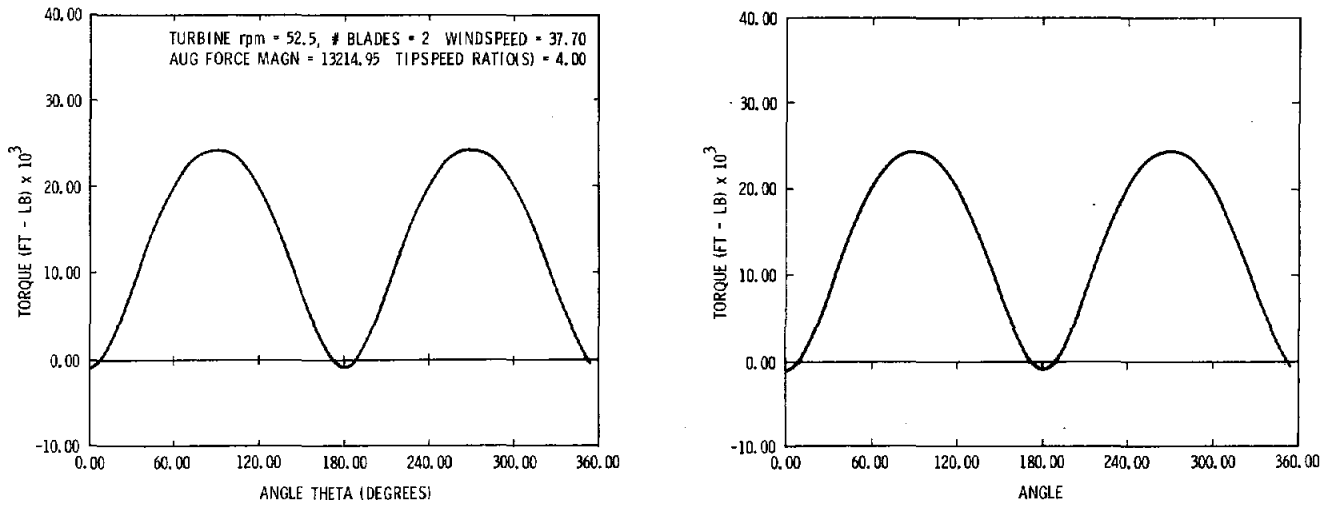
This procedure was repeated for 24 values of blade tipspeed-to-windspeed ratio (RW/V) in increments of one-half starting at RW/V = 1. Turbine rotational speeds of interest are 29.6, 37, 45.5, and 52.5 rpm for two- and three-bladed configurations with struts.

The period of the torque function is one-half rotation for two blades and one-third rotation for three blades. For accuracy and easy use with the FFT, we decided to use 32 increments per torque period. After calculating the torque for one period, we extended the results over 2^6 periods to obtain a periodic time series 2^{11} points long.

Similar calculations were made with the single Streamtube Model so that comparisons could be made between the two models.

The aerodynamic torque function is an even function of time because of upwind/downwind blade symmetry. Therefore, the coefficients turned out to be the real coefficients of the Fourier cosine series. Figures 4 and 5 show the graphs of the original functions and the series functions for the two-bladed configuration, with RW/V = 4.0, RW/V = 1.5, and rotational speed at 52.5 rpm. Table 1 lists an example of the coefficients obtained.

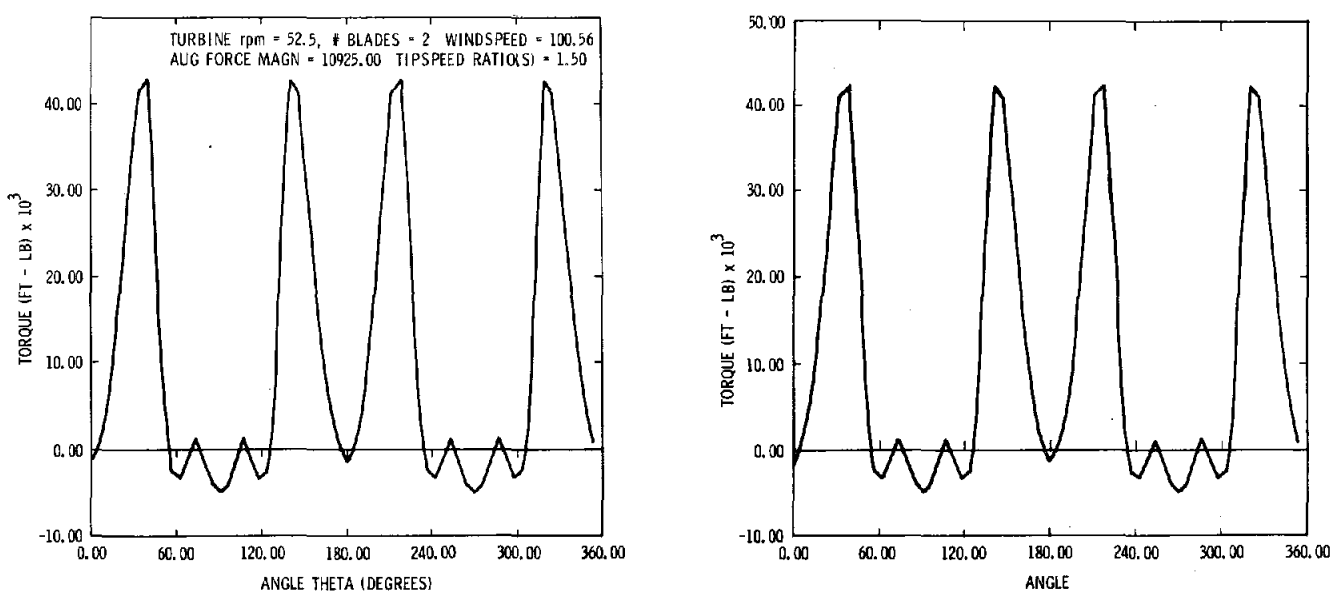
A good way to display the results is to plot the first five coefficients of the torque function vs RW/V. This is done for two blades and 52.5 rpm in Figure 6 and three blades in Figure 7. Thus, Figures 6 and 7 clearly show the behavior of the harmonic content of the torque.



a. Original Function

b. Series Function

Figure 4. Original Function vs Series Function of Aerodynamic Torque at RW/V = 4 for the 17-m VAWT Two-Bladed Configuration



a. Original Function

b. Series Function

Figure 5. Original Function vs Series Function of Aerodynamic Torque at RW/V = 1.5 for the 17-m VAWT Two-Bladed Configuration

Table 1

An Example of the Coefficients Obtained for the
17-m VAWT Two-Bladed Configuration

```
# BLADES= 2, RPM= 52.5
WIND SPEED= 25.8 MPH,  TSR=  4.0

THE FIRST  16 COEFFICIENTS, A0, A1, A2,..., OF THE COSINE SERIES,

A0+A1*COS(WT)+A2*COS(2WT)+...,

WHERE W IS THE ANGULAR FREQUENCY, ARE:
```

N	FREQUENCY (PER REV)	(HZ)	(RAD/SEC)	NTH FOURIER COEFF
0	0	.00	.00	13214.531250
1	2	1.75	11.00	-12361.179687
2	4	3.50	21.99	-1488.409668
3	6	5.25	32.99	-151.783295
4	8	7.00	43.98	-76.424118
5	10	8.75	54.98	-23.152176
6	12	10.50	65.97	-28.092754
7	14	12.25	76.97	25.743965
8	16	14.00	87.96	-31.312500
9	18	15.75	98.96	2.238873
10	20	17.50	109.96	4.724888
11	22	19.25	120.95	-3.912411
12	24	21.00	131.95	-6.950874
13	26	22.75	142.94	-2.941116
14	28	24.50	153.94	-3.972534
15	30	26.25	164.93	.484375

```
PLOTS ?
```

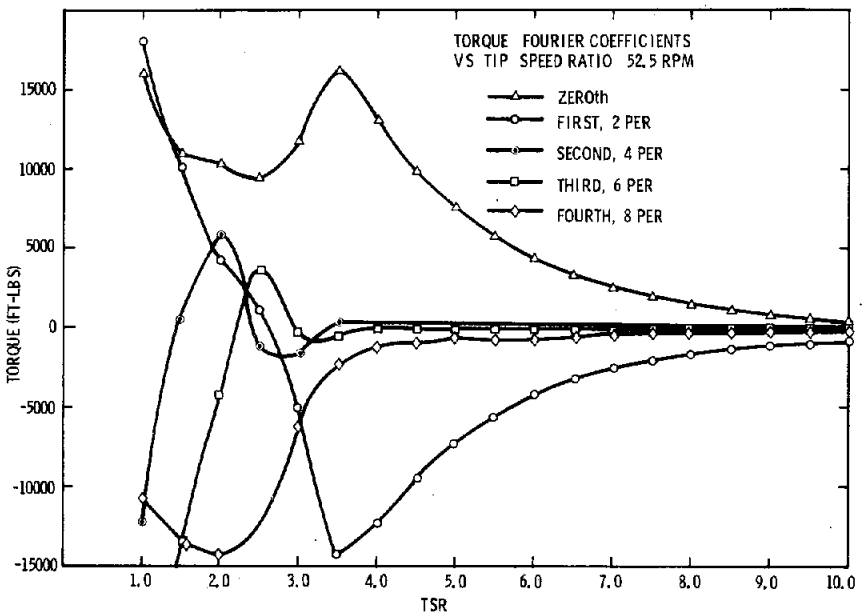


Figure 6. Plot Showing the First Five Coefficients of the Torque Function vs TSR, Two-Bladed Configuration

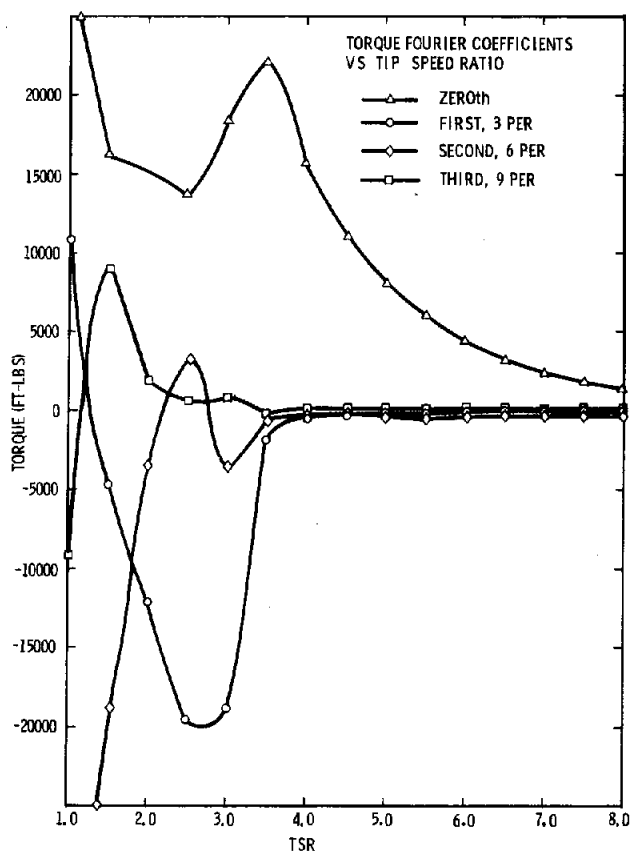


Figure 7. Plot Showing the First Five Coefficients of the Torque Function vs TSR, Three-Bladed Configuration

Finally, the coefficients of the Fourier cosine series were computed for the Single Streamtube Model and plotted in Figure 8 as a function of TSR along with the corresponding coefficients of the Multiple Streamtube Model so that the harmonic content of the two models can be compared.

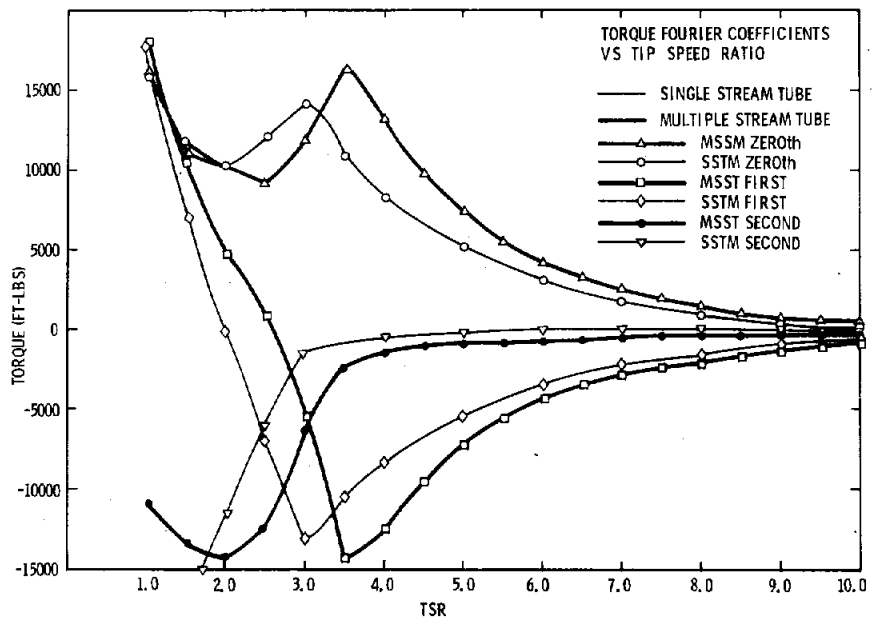


Figure 8. A Comparison of the Harmonic Content of the Single Streamtube Model and the Multiple Streamtube Model

Conclusions

The FFT method for computing Fourier coefficients is satisfactory for aerodynamic torque applications. The agreement between the aerodynamic torque function, TQ(t), and the series representation,

$$\frac{1}{2} a_0 + \sum_{n=1}^{15} a_n \cos (2\pi Wn) ,$$

is quite good. In particular, for 32 points in the fundamental period, and 2^6 periods, the FFT method produced coefficients that were acceptable in all cases.

One disadvantage of the FFT method described here is that the FFT was limited to data sets with 2^n points in the fundamental period. This limitation is not absolute since more general forms

of the FFT are available, but the FFT algorithms based on 2^n are the simplest and most easily available. The disadvantage became clear in the three-bladed case when the forces were to be added over the three blades.

Overall, the FFT method is recommended for use in similar situations.

References

[1] R. C. Reuter and M. H. Worstell, Torque Ripple in a Vertical Axis Wind Turbine, SAND78-0577 (Albuquerque, NM: Sandia Laboratories, 1978).

[2] J. H. Strickland, The Darrieus Turbine: A Performance Prediction Model Using Multiple Streamtubes, SAND75-0431 (Albuquerque, NM: Sandia Laboratories, 1975).

[3] L. Schwarz, Theorie des Distributions (Paris: Hermann & Cie, 1966).

[4] W. Ruden, Functional Analysis (New York: McGraw-Hill, 1973).

[5] E. O. Brigham, The Fast Fourier Transform (Englewood Cliffs, N.J.: Prentice-Hall, 1974).

[6] J. W. Cooley, and J. W. Tukey, "An Algorithm for Machine Calculation of Complex Fourier Series," Math Computation 5 (April 1965): 297-301.

[7] P. C. Klimas and R. E. Sheldahl, Four Aerodynamic Prediction Schemes for Vertical-Axis Wind Turbines: A Compendium, SAND78-0014 (Albuquerque, NM: Sandia Laboratories, 1978).

APPENDIX A

FFT FORTRAN IV Subroutine

```
FTN4,L
      SUBROUTINE FFT(FR,FI,K)
C
C  FFT FAST FOURIER TRANSFORMS COMPLEX
C  DATA IN FR(REAL) AND FI(IMAGINARY)
C  ARRAYS. THE NUMBER OF POINTS TO BE
C  TRANSFORMED MUST BE N=2**K.
C
      DIMENSION FR(1),FI(1)
      PI=3.14159265
      N = 2**K
      MR = 0
      NM1=N-1
      DO 20 M=1,NM1
      L = N
   10 CONTINUE
      L=L/2
      IF(MR+L.GT.NM1) GO TO 10
      MR = MOD(MR,L) + L
      IF(MR.LE.M)GO TO 20
      MP1 = M + 1
      MRP1 = MR + 1
      TR = FR(MP1)
      FR(MP1) = FR(MRP1)
      FR(MRP1) = TR
      TI = FI(MP1)
      FI(MP1) = FI(MRP1)
      FI(MRP1) = TI
   20 CONTINUE
      L = 1
   30 CONTINUE
      IF(L.GE.N) RETURN
      ISTEP = 2*L
      EL = L
      DO 40 M=1,L
      A = PI*FLOAT(1-M)/EL
      WR=COS(A)
      WI = SIN(A)
      DO 40 I=M,N,ISTEP
      J = I + L
      TR = WR*FR(J) - WI*FI(J)
      TI = WR*FI(J) + WI*FR(J)
      FR(J) = FR(I) - TR
      FI(J) = FI(I) - TI
      FR(I) = FR(I) + TR
      FI(I) = FI(I) + TI
   40 CONTINUE
      L = ISTEP
      GO TO 30
      END
      END$
**** LIST END ****
@
```

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