VABS: Going Beyond Linear Elastic Cross-Sectional Analysis

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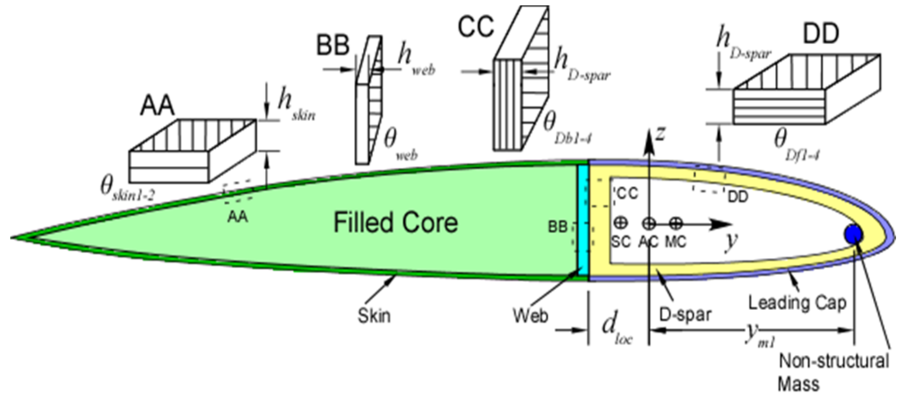
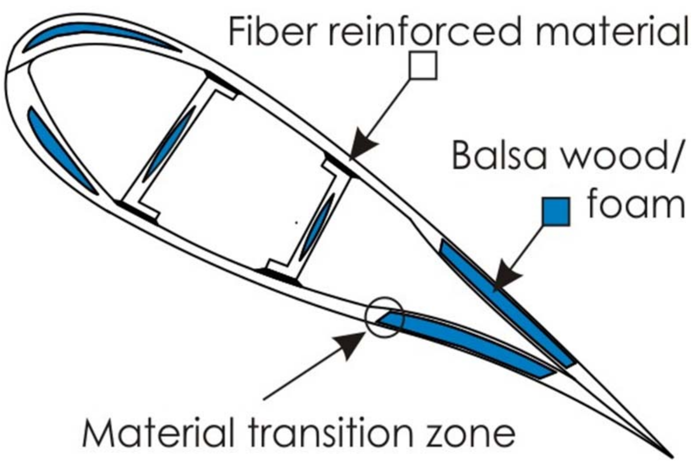
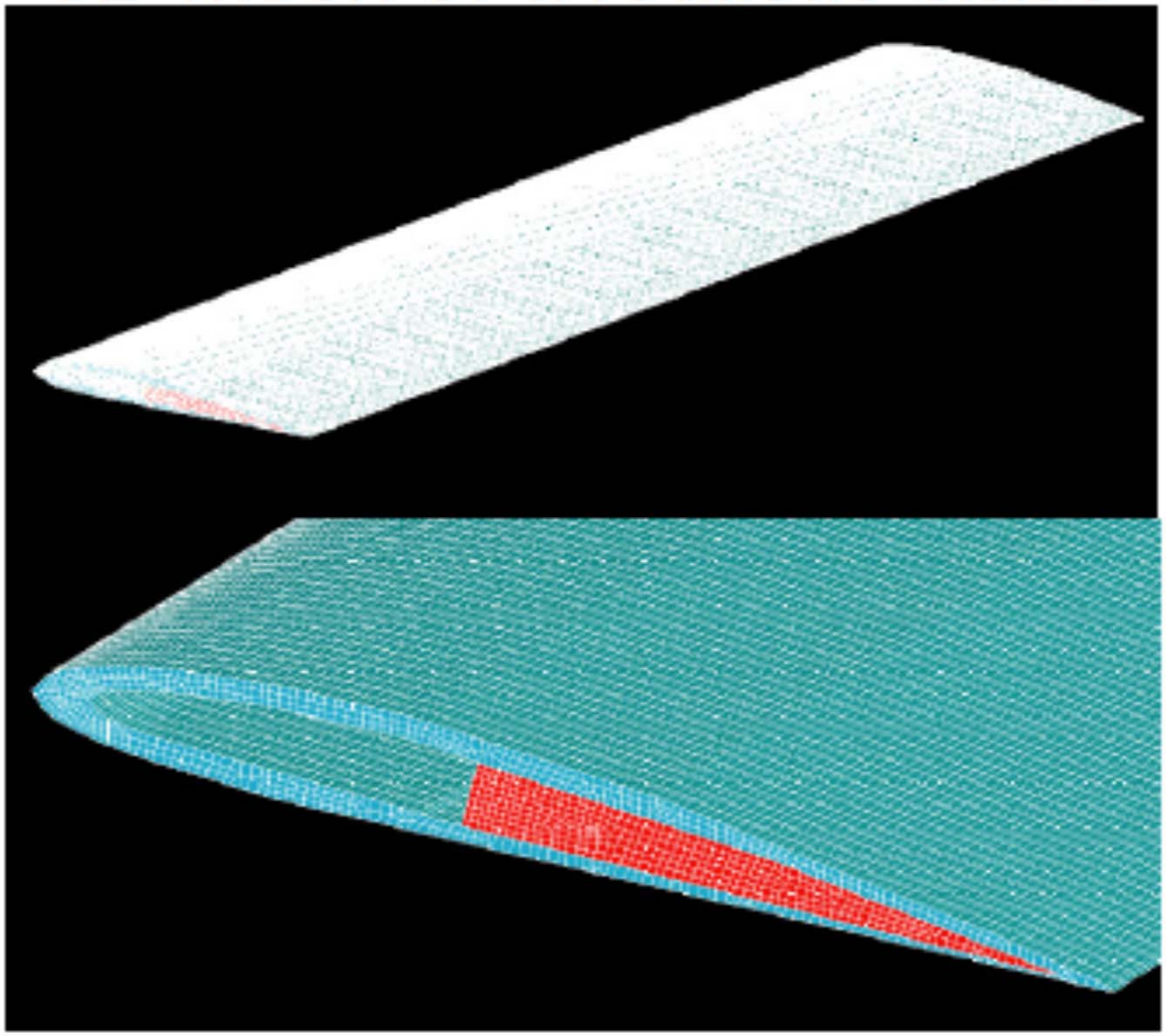
Engineering Challenges





Engineering Challenges (cont.)



Millions to billions DOFs!

Not suitable for preliminary design

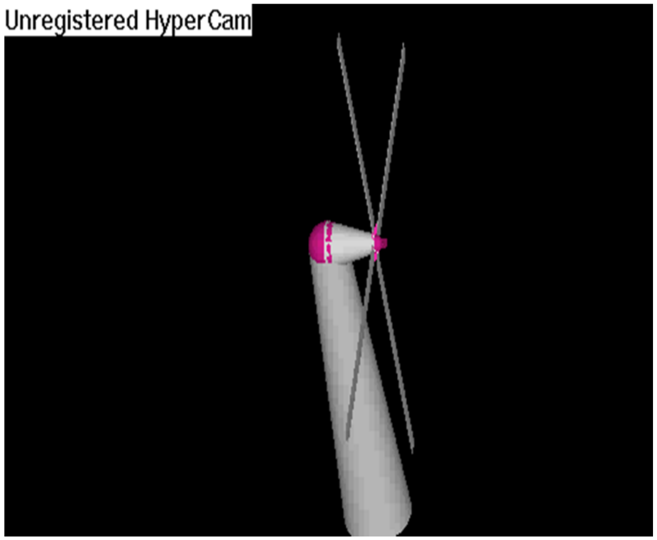
Not suitable for aeroelastic analysis

Unnecessary waste of engineers' time and computing resources

Engineering Challenges (cont.)

- Testing is difficult and expensive, particularly for large blades
- State of the art: multibody dynamic simulation integrating both aerodynamic and structural concerns
- Success of this simulation relies on accurate blade modeling to link structural details with blade properties

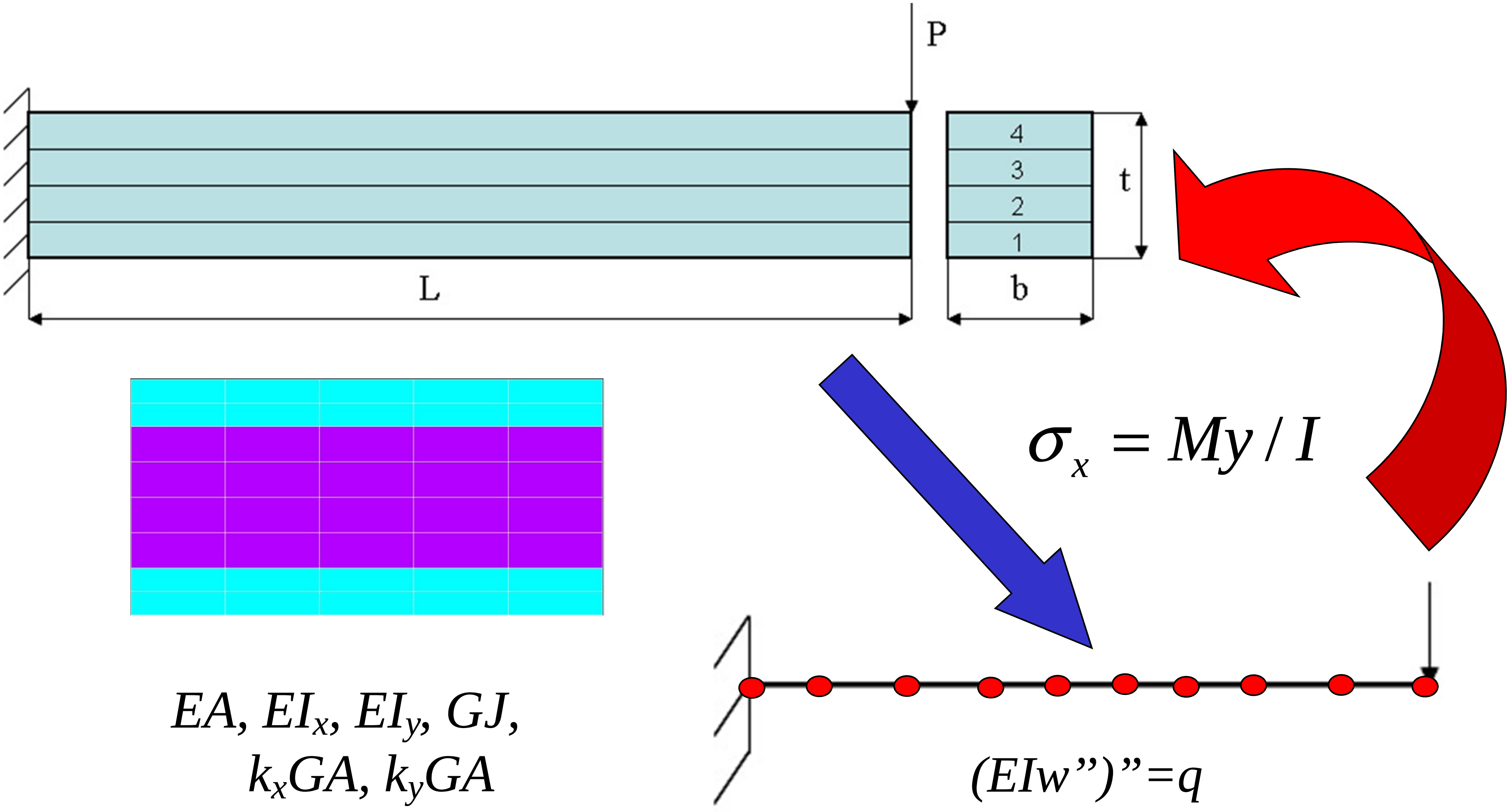




Beam Theory

- Beam theory provides an effective solution to avoid prohibitive full 3D analysis
- Has a rich history of 400+ years: Leonardo da Vinci, Galileo Galilei, Bernoulli brothers, Leonhard Euler, etc.
- Three basic elements of a beam theory
  - Ways to evaluate beam properties: *EA*, *EI*, *GJ*, etc.
  - A closed set of 1D differential equations to predict structural behavior $(EIw'')''=q$
  - Relations to recover 3D fields in terms of beam variables: $\sigma_x = My/I$
- Mainly based on ad hoc assumptions: c/s remain planar & normal, uniaxial stress, etc.

Beam Theory (cont.)



$$\sigma_x = My / I$$

EA, EI_x, EI_y, GJ,
k_xGA, k_yGA

$(EIw'')'' = q$

Beam Theory (cont.)

- For an isotropic, homogeneous beam
  - E-B model:

$$F_1 = EA\,\gamma_{11}$$
$$M_1 = GJ\,\kappa_1$$
$$M_2 = EI_{22}\,\kappa_2$$
$$M_3 = EI_{33}\,\kappa_3$$

$$\begin{Bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{bmatrix} EA & 0 & 0 & 0 \\ 0 & GJ & 0 & 0 \\ 0 & 0 & EI_{22} & 0 \\ 0 & 0 & 0 & EI_{33} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}$$

  - Timoshenko model:

$$F_1 = EA\,\gamma_{11}$$
$$F_2 = k_2 GA\,2\gamma_{12}$$
$$F_3 = k_3 GA\,2\gamma_{13}$$
$$M_1 = GJ\,\kappa_1$$
$$M_2 = EI_{22}\,\kappa_2$$
$$M_3 = EI_{33}\,\kappa_3$$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{bmatrix} EA & 0 & 0 & 0 & 0 & 0 \\ 0 & k_2GA & 0 & 0 & 0 & 0 \\ 0 & 0 & k_3GA & 0 & 0 & 0 \\ 0 & 0 & 0 & GJ & 0 & 0 \\ 0 & 0 & 0 & 0 & EI_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & EI_{33} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ 2\gamma_{12} \\ 2\gamma_{13} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}$$

Beam Theory (cont.)

- Diagonal stiffness matrix, only possible for highly regular sections with restrictive choices of reference line, not valid for real blades, convenient for linear static analysis
- This convenience lost for other analyses
  - Nonlinear analysis/composites: deformation coupled
  - Dynamic analysis: mass center and principal inertial axes might be a more convenient choice for reference line
  - Aeroelastic analysis: may choose aerodynamic center
- Positional & directional offsets are needed to transform the stiffness matrix
- Allows analysts to choose any convenient reference line

Beam Theory (cont.)

- Structural properties: reproduce the 3D strain energy in a 1D beam model, stiffness matrix
  - Classical model: Euler-Bernoulli

$$\begin{Bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}$$

  - Refined model: Timoshenko

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ 2\gamma_{12} \\ 2\gamma_{13} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}$$

  - 1D beam analysis for composite blades should accept fully

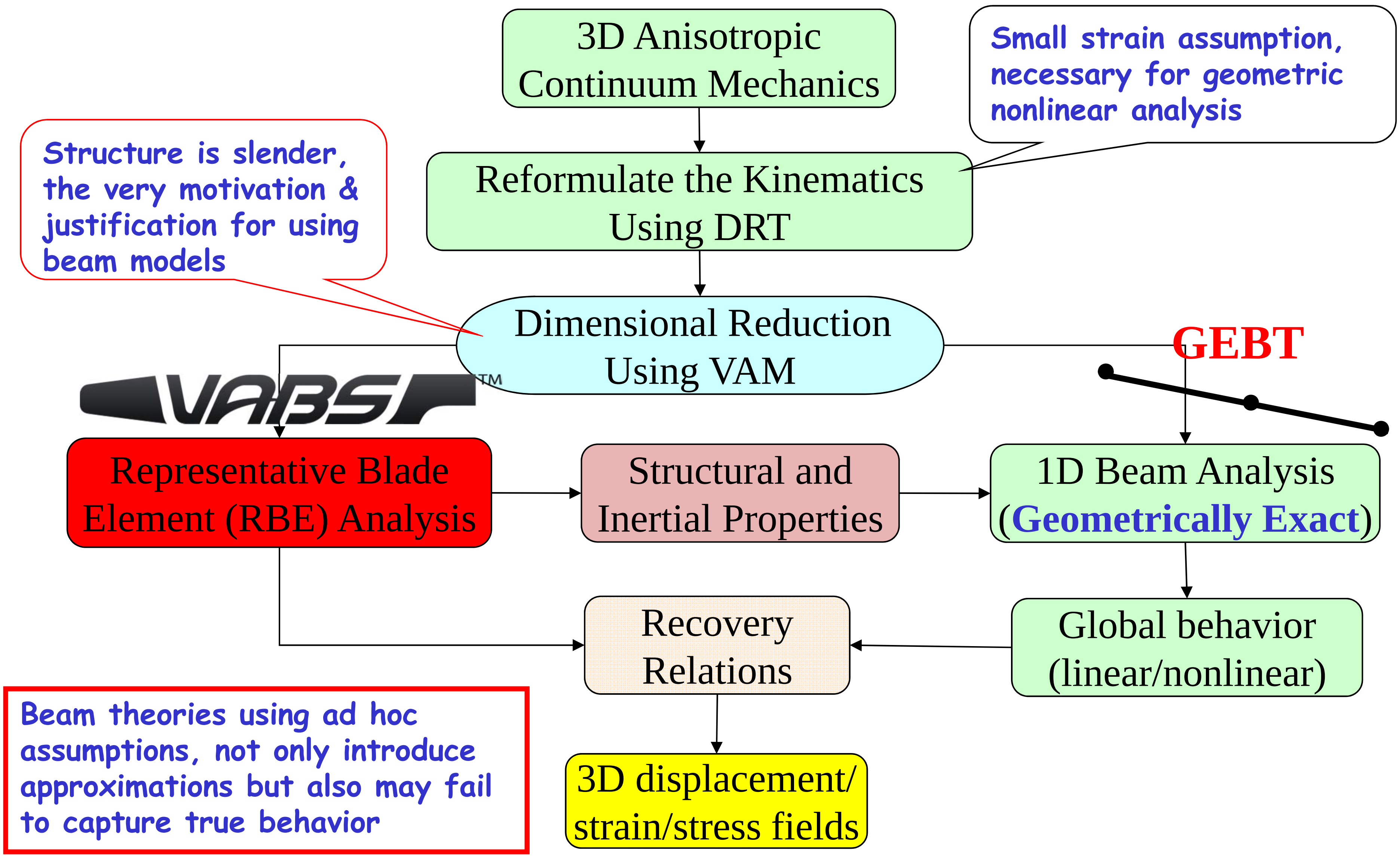
Beam Theory (cont.)

- Inertial properties: reproduce 3D kinetic energy using a beam model, mass matrix

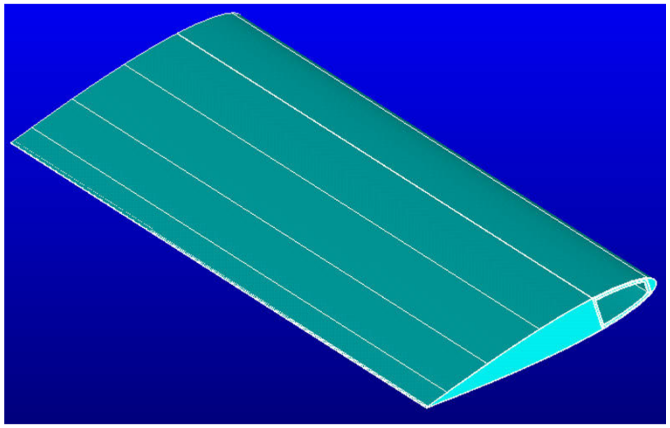
$$\mathcal{K} = \frac{1}{2} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{Bmatrix}^T \begin{bmatrix} \mu & 0 & 0 & 0 & \mu x_{m3} & -\mu x_{m2} \\ & \mu & 0 & -\mu x_{m3} & 0 & 0 \\ & & \mu & \mu x_{m2} & 0 & 0 \\ & & & i_{22} + i_{33} & 0 & 0 \\ & symmetric & & & i_{22} & -i_{23} \\ & & & & & i_{33} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{Bmatrix}$$

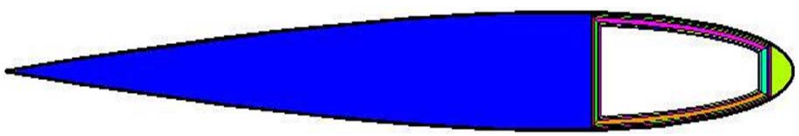
- Recover 3D fields: all six components of the stress/strain tensors might be significant
- 1D beam analysis remains same as isotropic blades. Only difference for composite blades is how to bridge 3D model with 1D beam model

The Basic Ideas of VABS

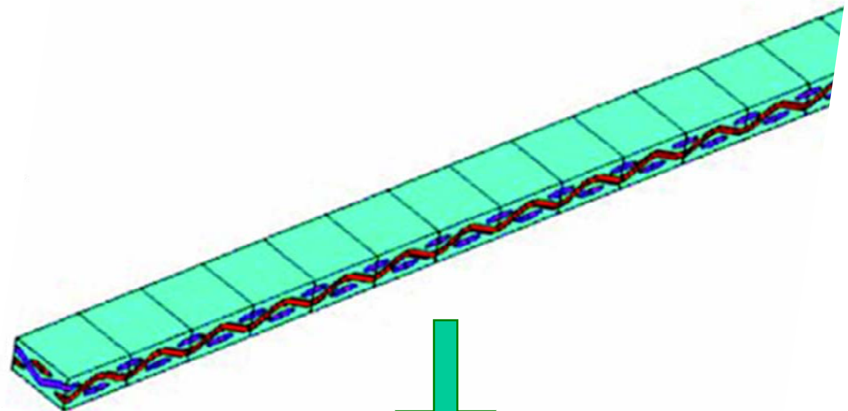


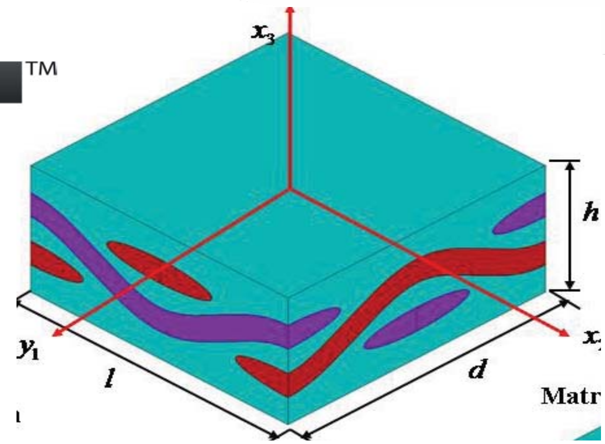
The Basic Ideas of VABS (cont.)





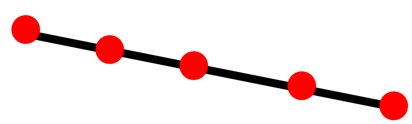
2D RBE





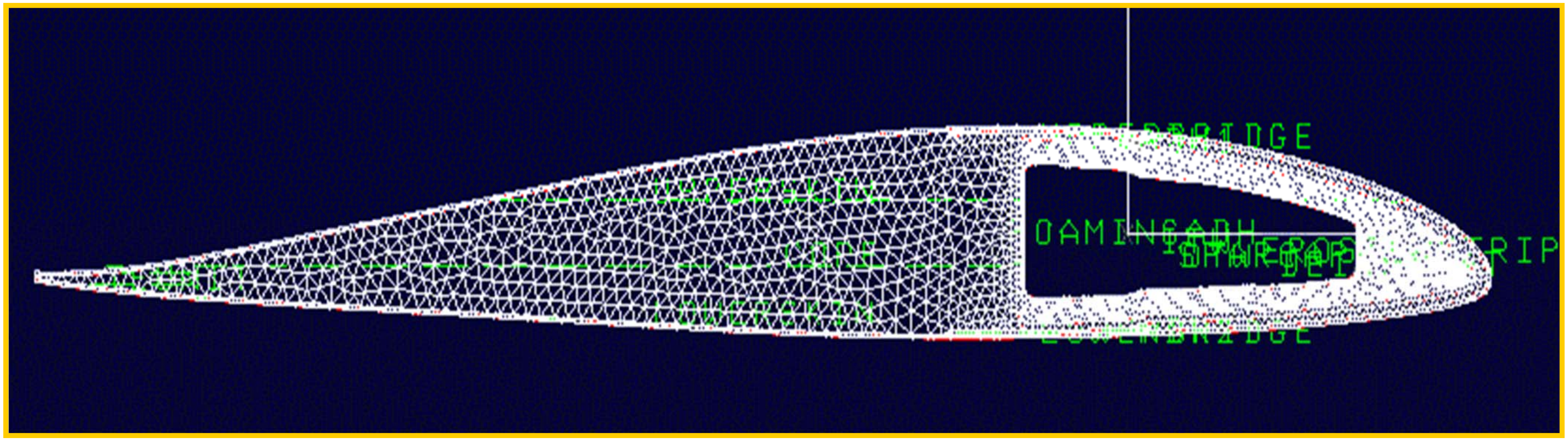
3D RBE

+



GEBT

What VABS Can Do for You?



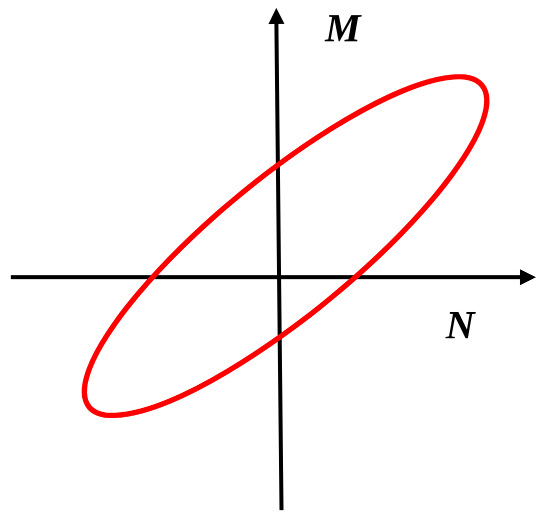
- **VABS** takes a finite element discretization of RBE including geometry and material as input to calculate blade properties, which are needed for any beam analysis code to predict global behavior. VABS also recovers 3D displacements/strains/stresses over the RBE: a link between 3D and 1D
- **VABS** can be used independently for **structural design of composite blades** (topology and material): e.g., maximize twist-bend coupling while maintaining other properties fixed
- **VABS** rigorously models composite blades with no additional cost to 1D beam analysis, enabling designers to go beyond “black aluminum”

VABS Outputs

- Inertial properties: mass matrix, mass center, principal inertial axes
- Structural properties
  - Classical stiffness/flexibility matrices
  - Neutral axes (tension center)
  - Timoshenko stiffness/flexibility matrices
  - Shear center (elastic center)
- Accurate 3D fields: displacement (3 components), strain (6 components), stress (6 components)
- Multiphysical (thermal, mechanical, electric, and magnetic) properties/behavior: environmental effects

Possible Uses of VABS

- Obtain blade properties as inputs for blade analyses using beam theory (static, dynamic, buckling, etc)
- Recover accurate 3D fields without the cost of expensive 3D FEA
- Design distortion free laminate using VABS thermoelastic capability
- Analyze actuating or sensing of smart materials using VABS multiphysics capability
- Predict fatigue life by coupling VABS and 1D beam dynamic analysis
- Create design envelopes in terms of stress resultants
- Could be used as standalone code or an integrated module
  - Integrate VABS with an optimizer for design tradeoffs
  - Integrate VABS with statistics tools to propagate statics of material properties & geometry to blade properties and to blade behavior

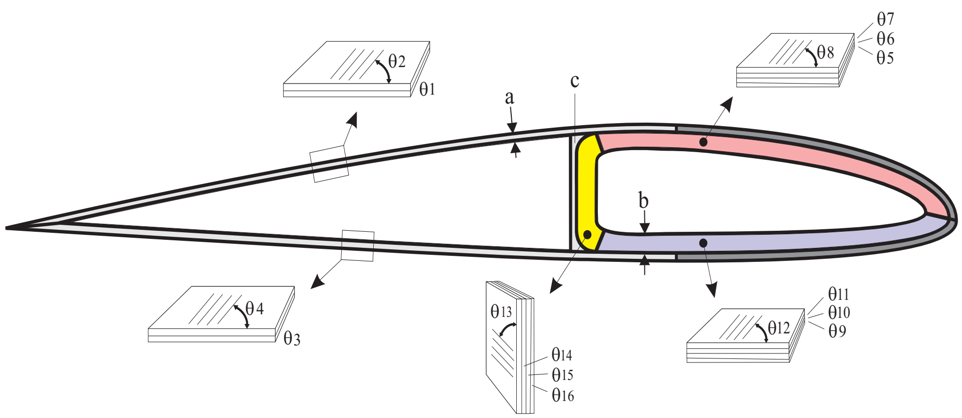


VABS for Cross-Sectional Design

There's Plenty of Room at the Bottom

An Invitation to Enter a New Field of Physics

- *Richard P. Feynman* (12/29/1959)

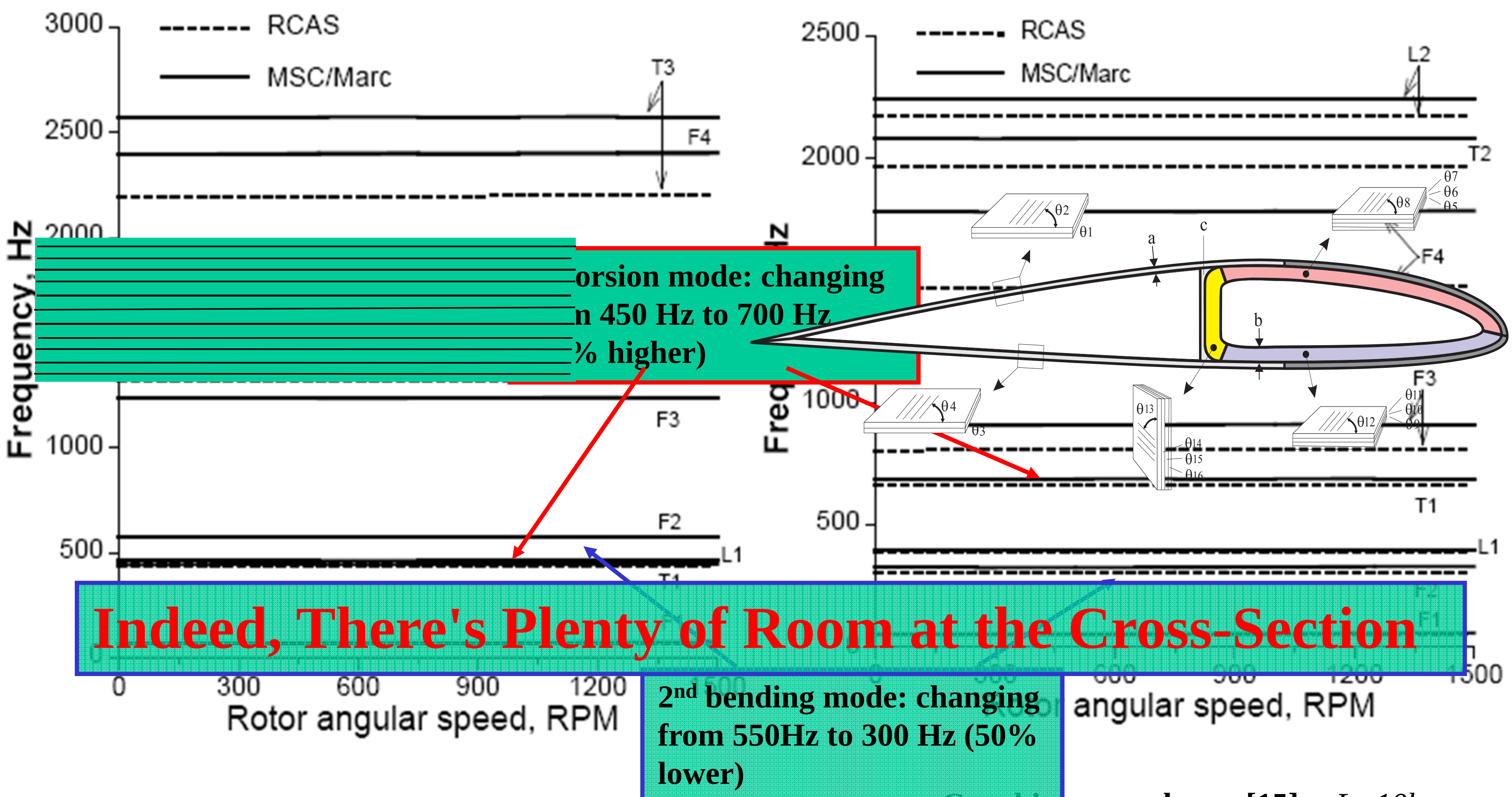


There's Plenty of Room at the Cross-Section

An Invitation to Enter a New Field of Blade Design

- Wenbin Yu (05/18/2010)

VABS for Cross-Sectional Design (cont.)

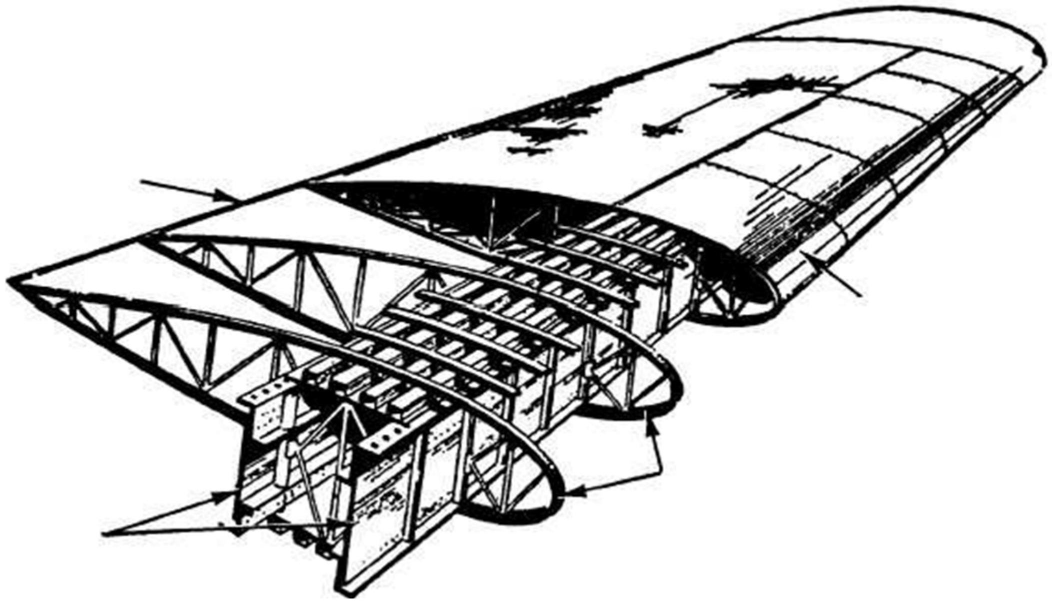


Graphite-epoxy beam $[0]_{24}$ *L=10b*

Graphite-epoxy beam $[15]_{24}$ *L=10b*

Source: *Yeo, H.; Truong, K.V.; Ormiston, R.A.: "Assessment of 1D Versus 3D Methods for Modeling Rotor Blade Structural Dynamics", SDM 2010, AIAA Paper #2010-3044*

VABS for Cross-Sectional Design (cont.)

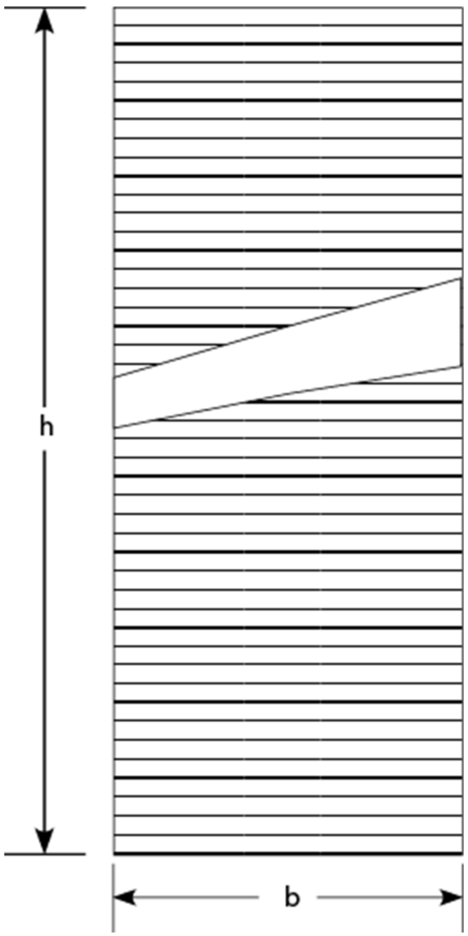


Complexity mainly resulted from a piecemeal approach for different functions and failure modes

Large turbine blades might get more complex if we blindly follow the past practice of aerospace industry

An integrative and holistic approach for blade design: based on rigorous science and engineering

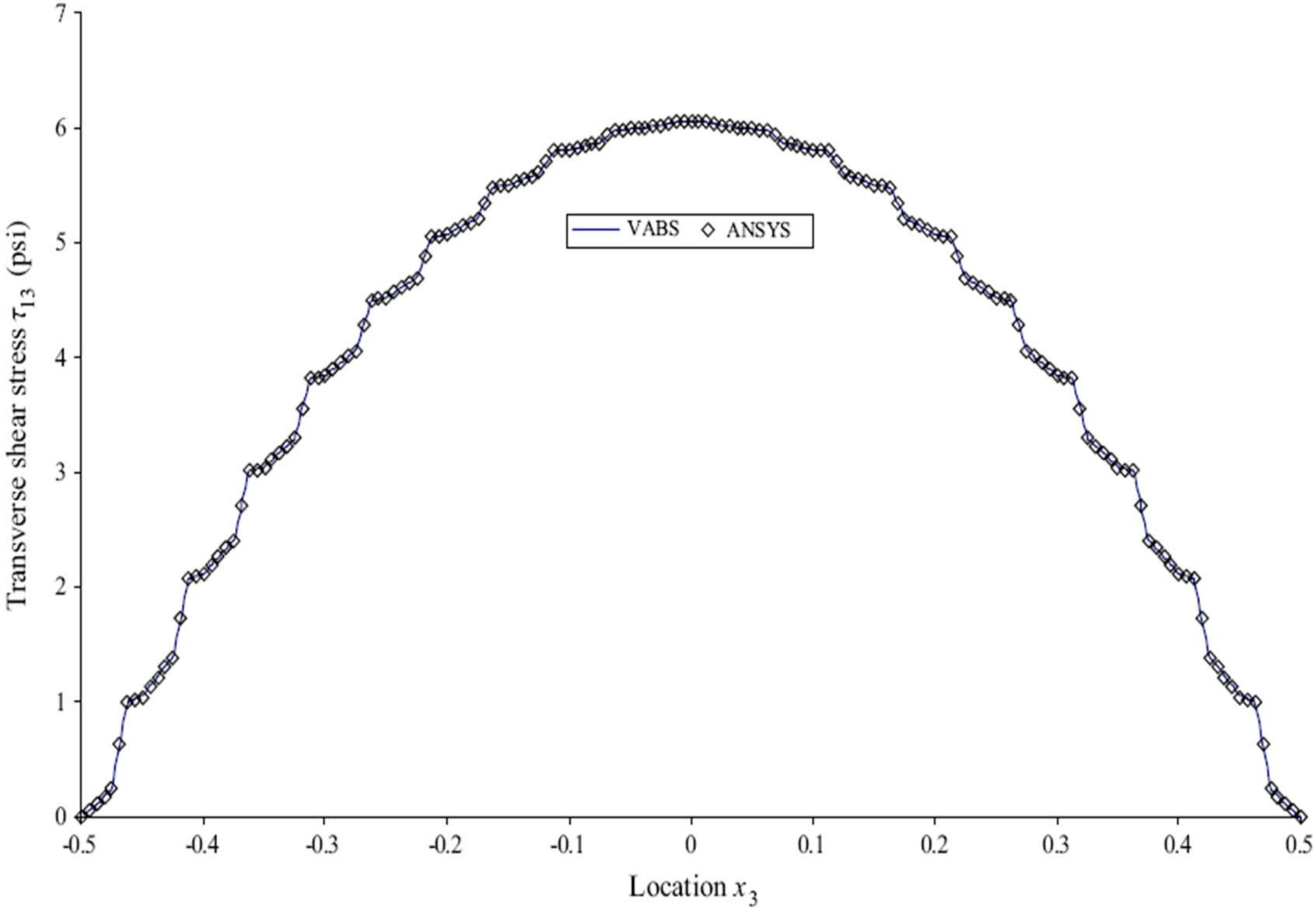
Rectangular Composite Beam



Cantilever composite rectangular beam:

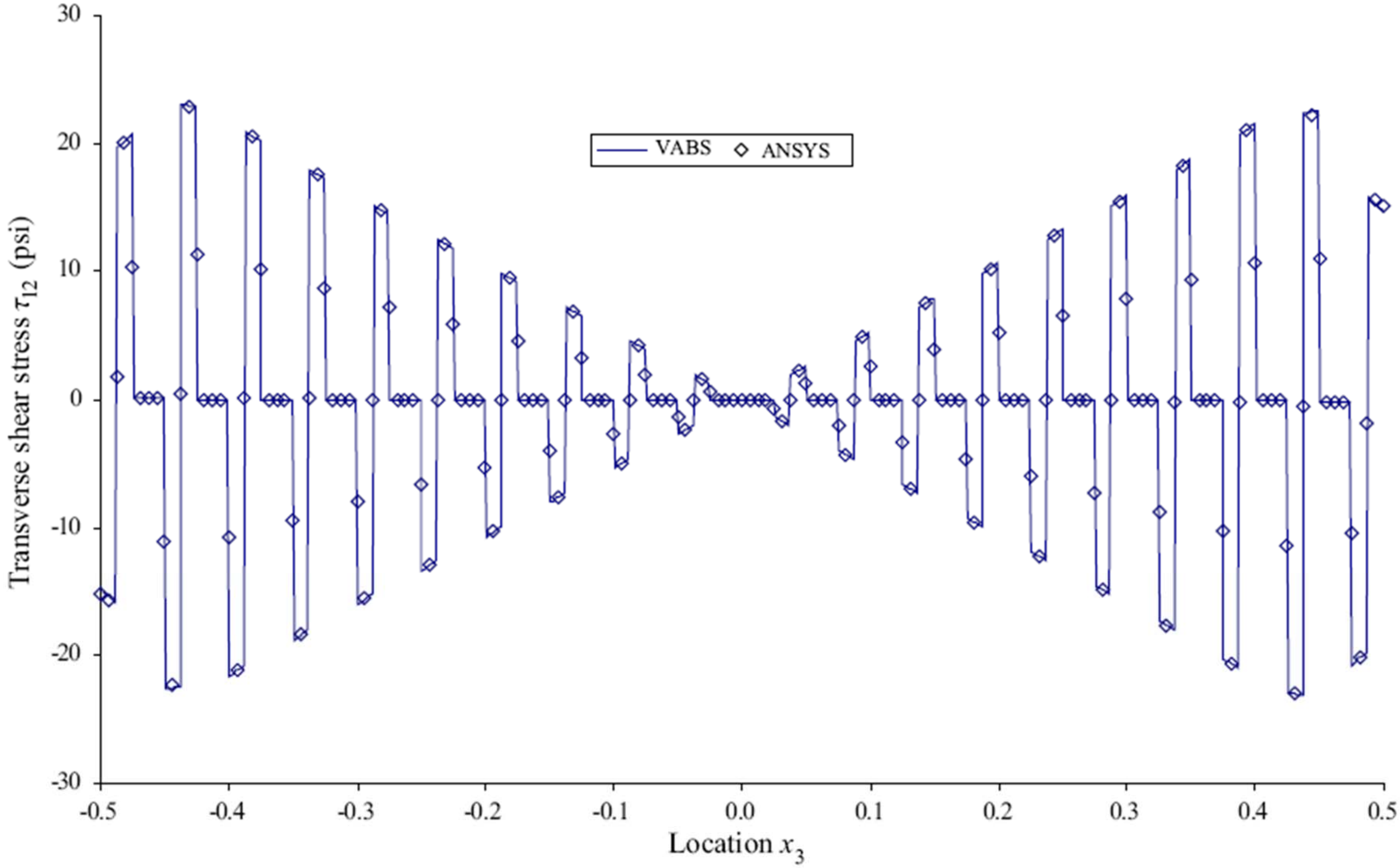
- **Layup:** $[(-45/+45/0/90)10]_s$
- ***b*=0.25 in., *h*=1 in., *L*=5 in.**
- **Shear force applied at the tip:**
- **ANSYS 3D FEA uses 25,600 brick elements; runs about one hour on a PC**
- **VABS (640 quadrilateral elements) less than 0.1 second; 1D solution can be obtained analytically**

Rectangular Composite Beam (cont.)



Transverse shear stress τ_{13} at mid-span and $x_2 = 0$

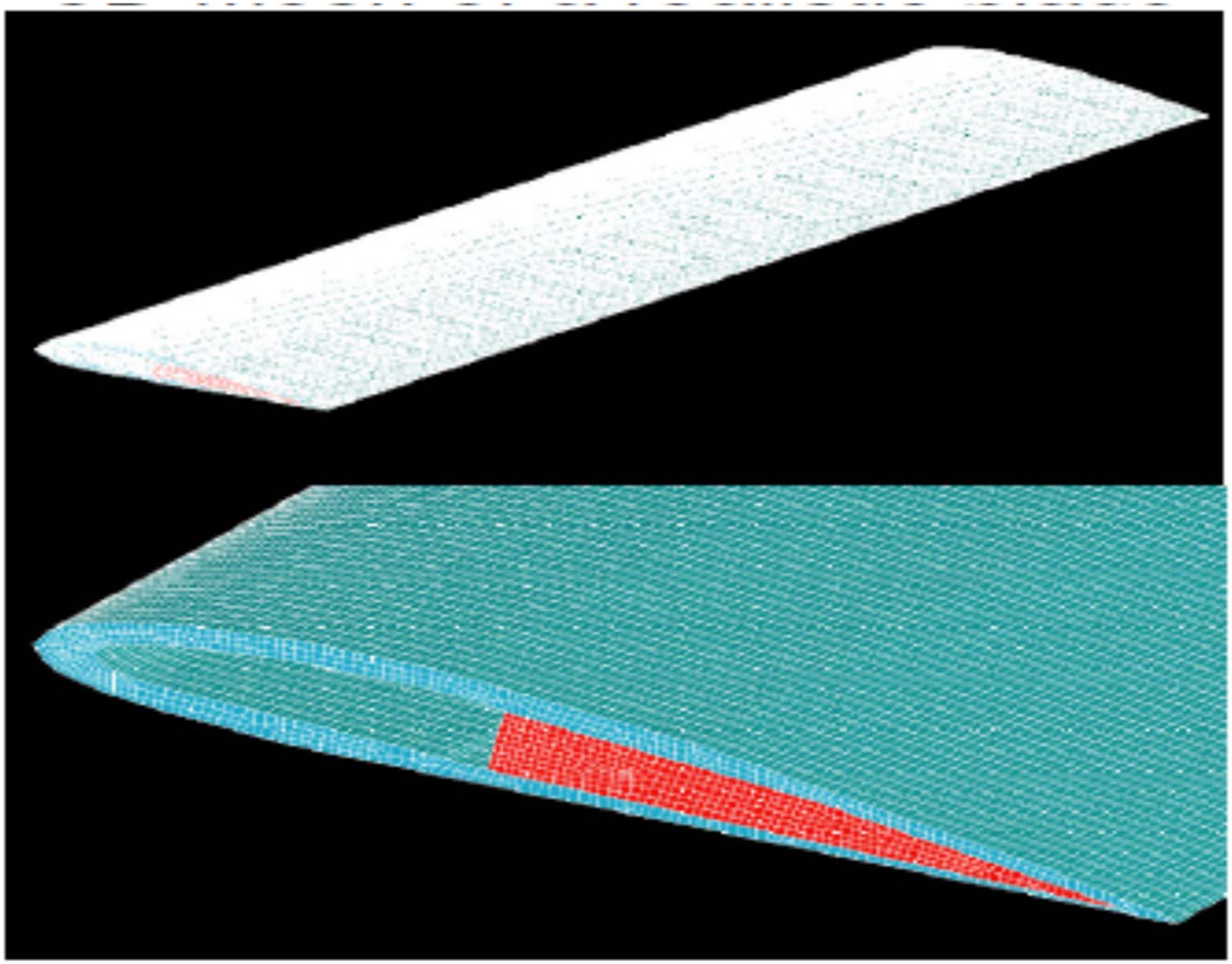
Rectangular Composite Beam (cont.)



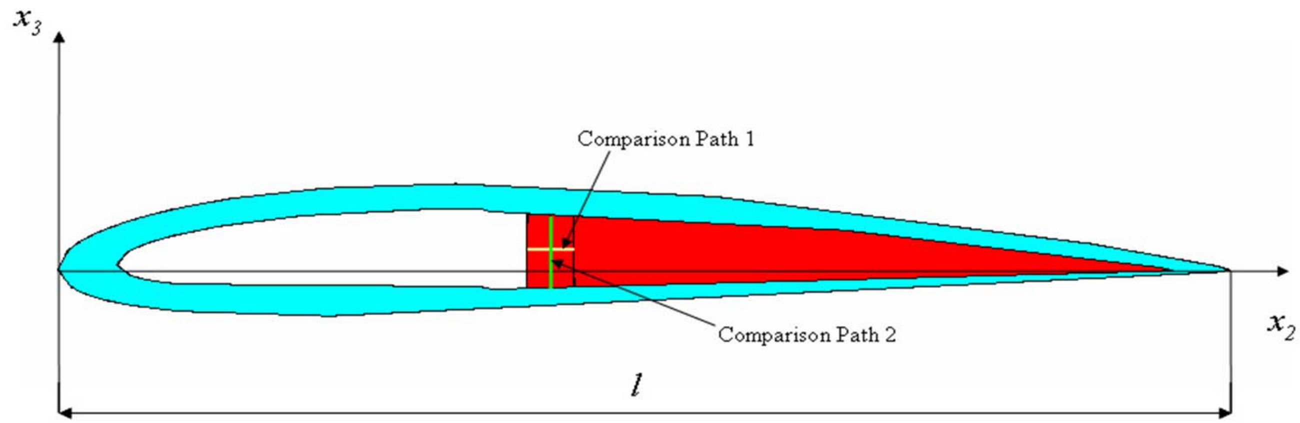
Transverse shear stress τ_{12} at mid-span and $x_2 = 0$

Realistic Composite Blade

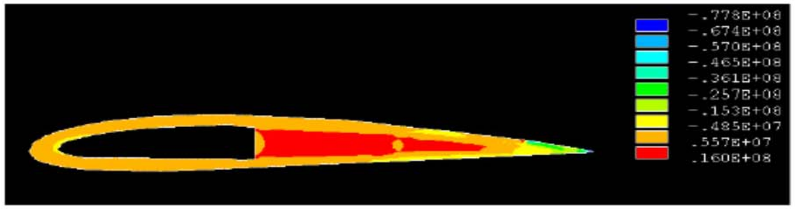
A realistic rotor blade under 100 degree C temperature change



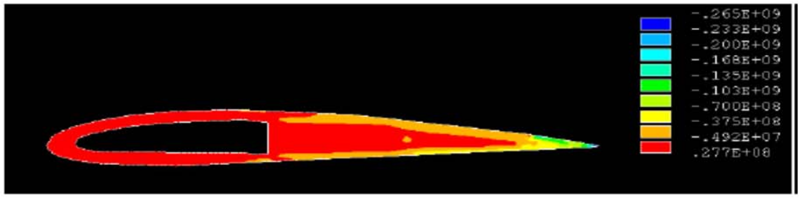
Realistic Composite Blade (cont.)



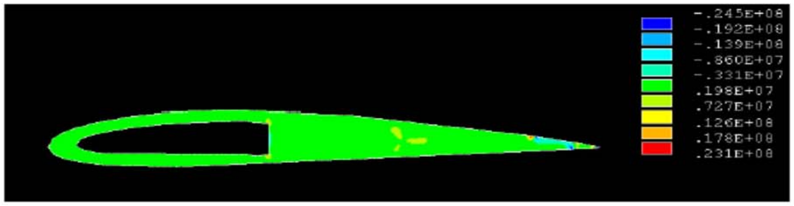
- σ_{11}



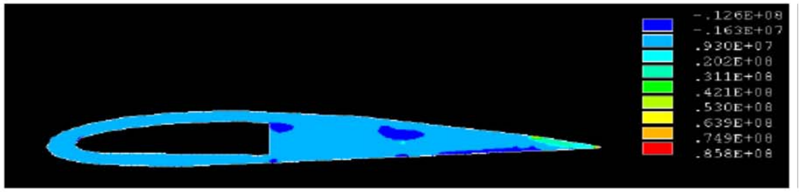
- σ_{22}



- σ_{33}

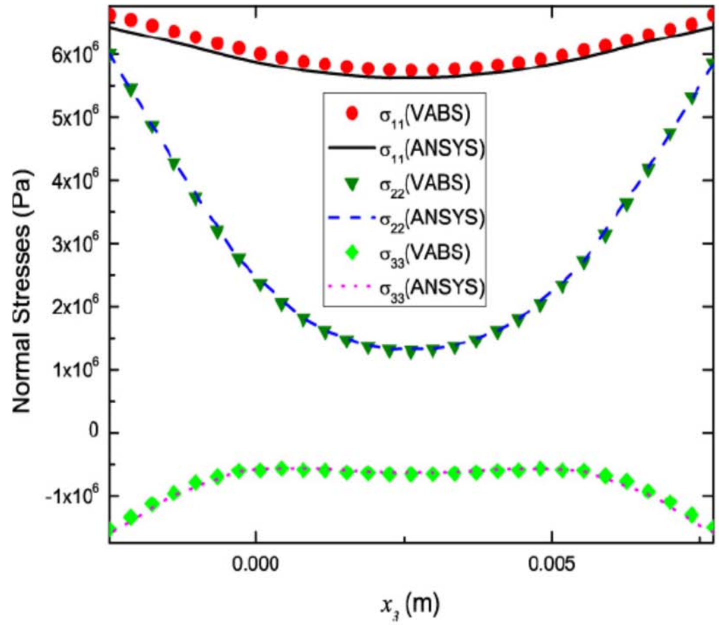


- σ_{23}

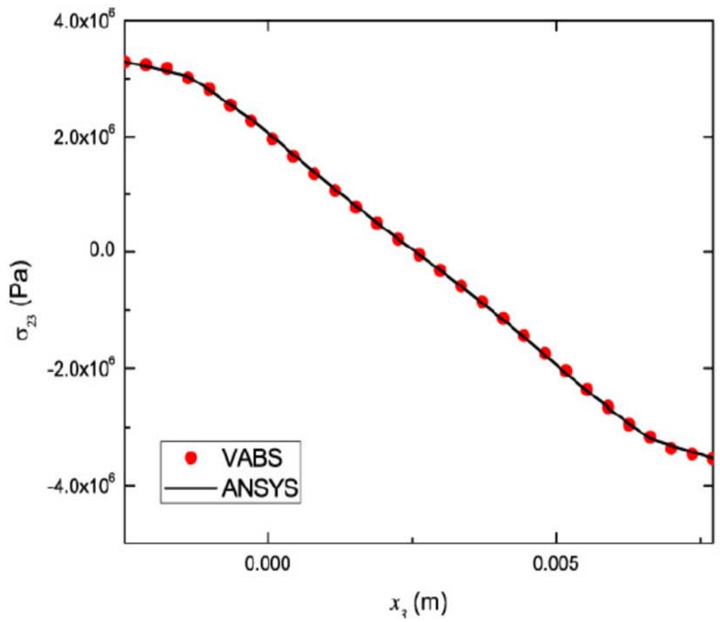


Realistic Composite Blade (cont.)

- Normal Stresses

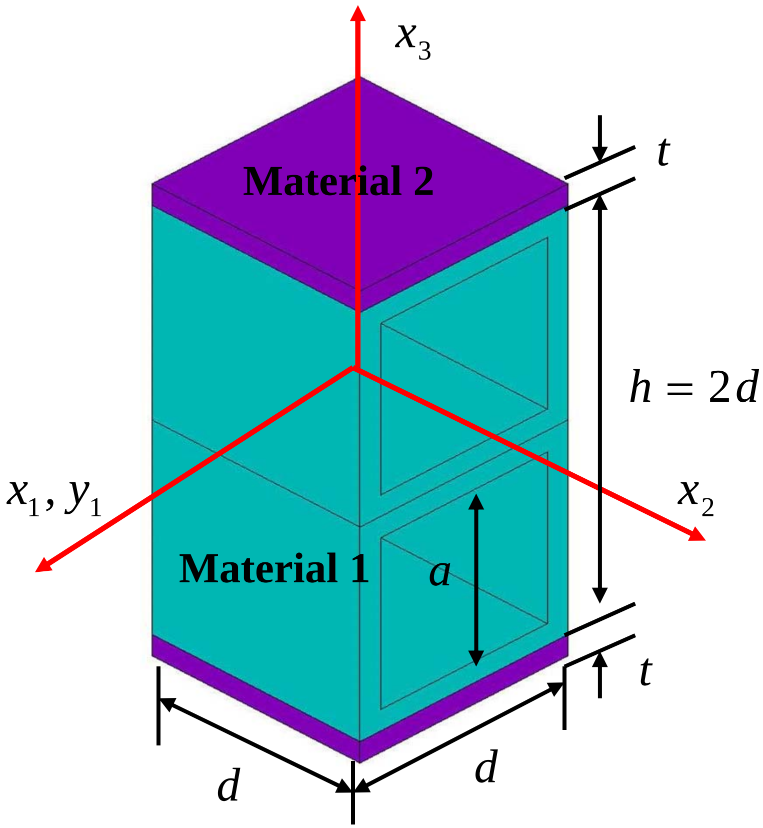


- σ_{23}



	ANSYS 3D	VABS
Element Type	SOLID186	8-noded quadrilateral
Number of Elements	362,408	2,459
Number of Nodes	1,638,866	7,965
Running Time	3h 5min 23s	**1.6s + 1.3s**

Heterogeneous Beam



Geometric variables:

$$t = 0.1\,\text{m} \quad h = 2d = 3.0\,\text{m} \quad a = 1.0\,\text{m}$$

Material 1:

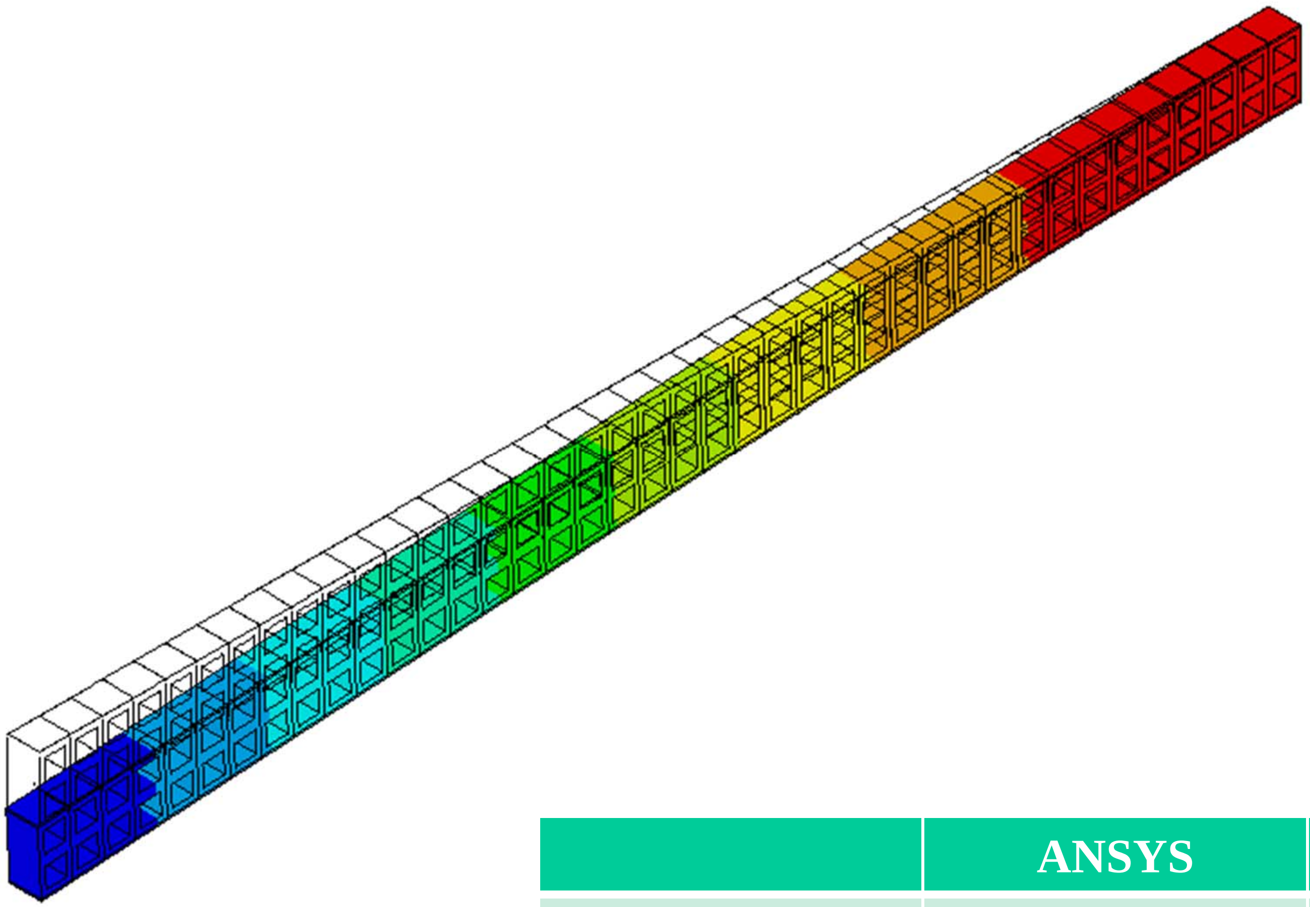
$$E_1 = 3.5\,\text{GPa} \quad \nu_1 = 0.34$$

Material 2:

$$E_2 = 70\,\text{GPa} \quad \nu_1 = 0.34$$

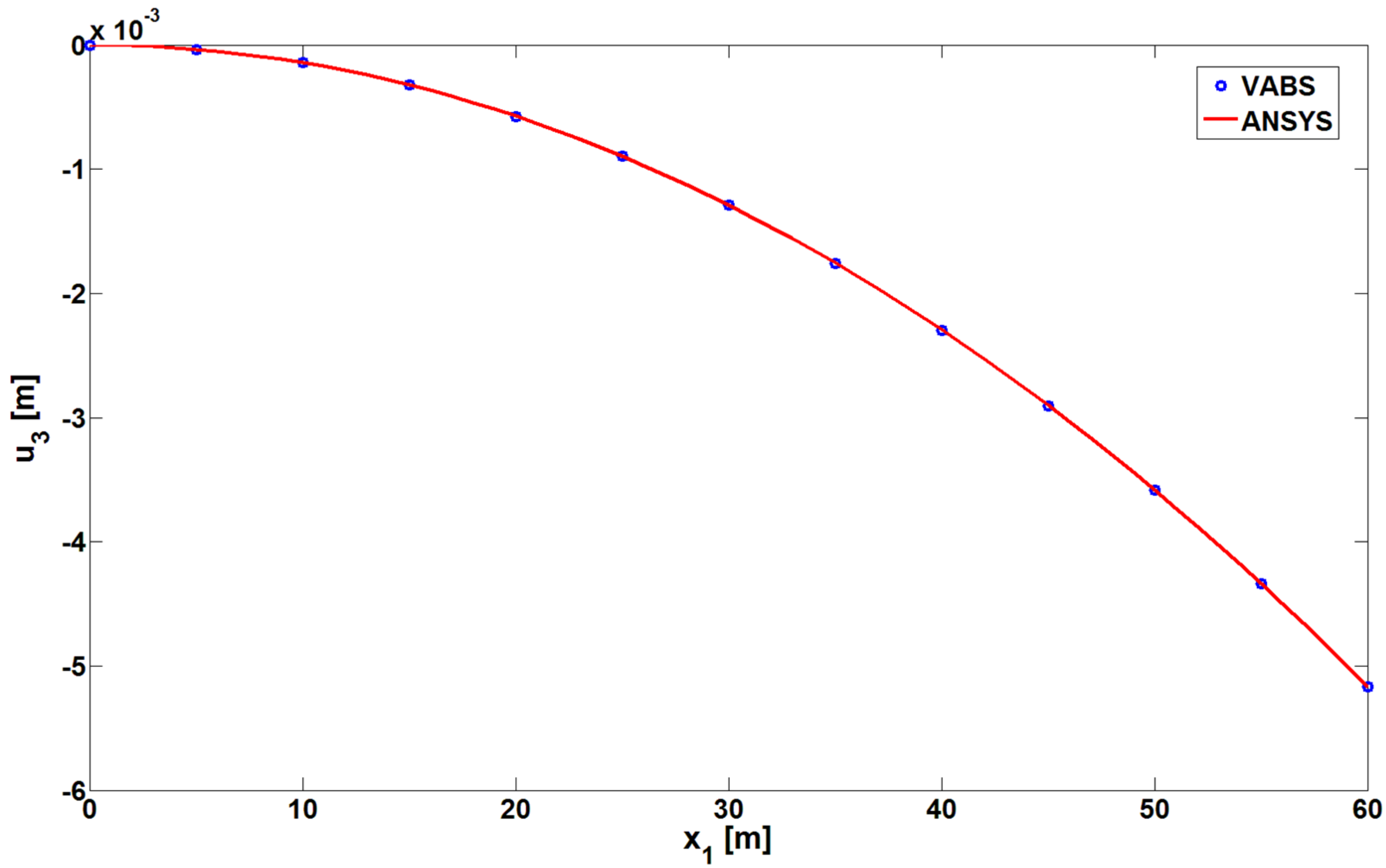
	$\bar{d}_{11}\left(\text{N}\right)$	$\bar{d}_{22}\left(\text{N.m}^2\right)$	$\bar{d}_{33}\left(\text{N.m}^2\right)$	$\bar{d}_{44}\left(\text{N.m}^2\right)$
VABS	2.66E+10	0.72E+9	5.58E+10	4.98E+9

Heterogeneous Beam (cont.)

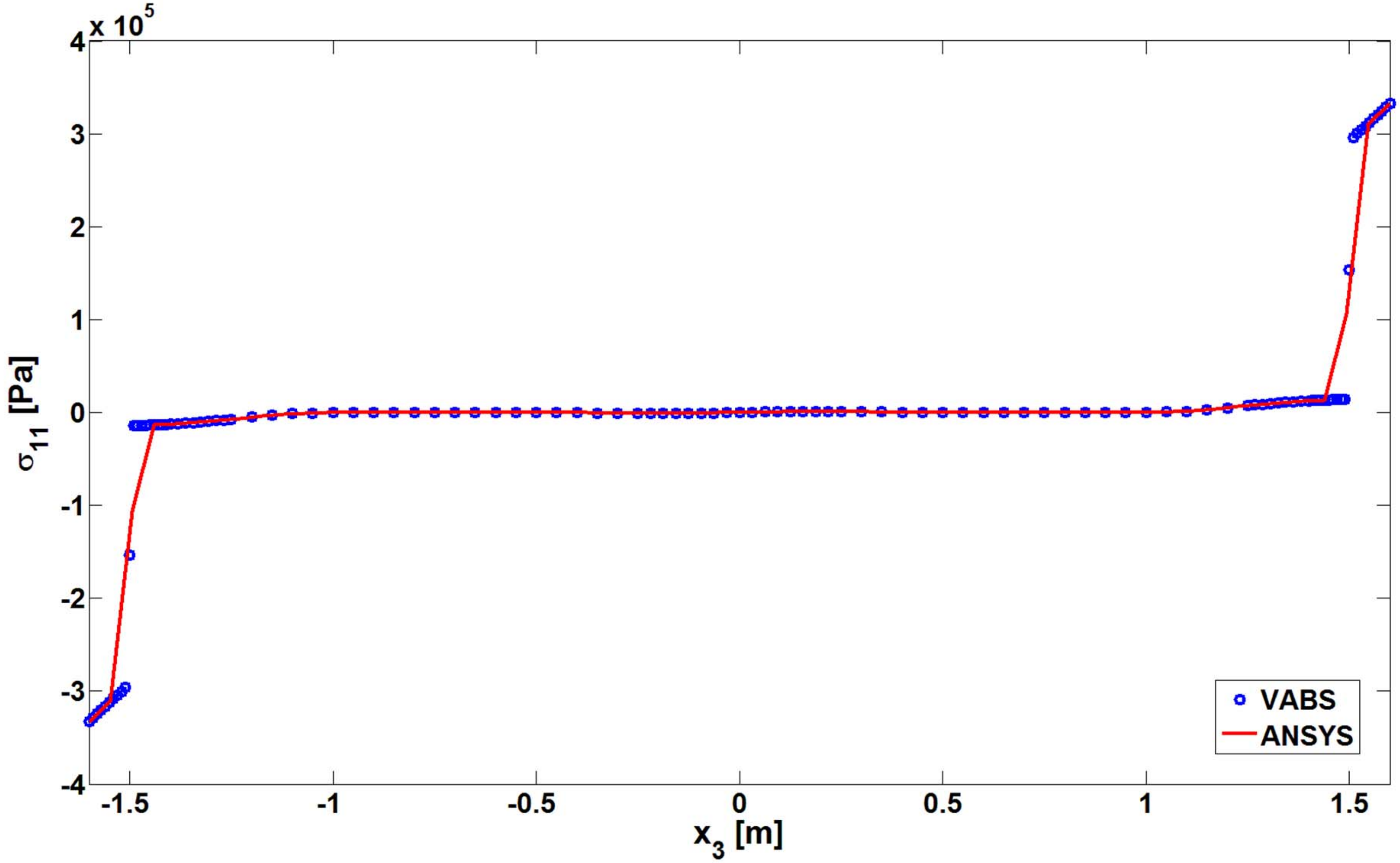


	ANSYS	VABS
Total element	736,000	18,400
Solved equation	9,599,700	239,994
Run time	11 hours	35 min

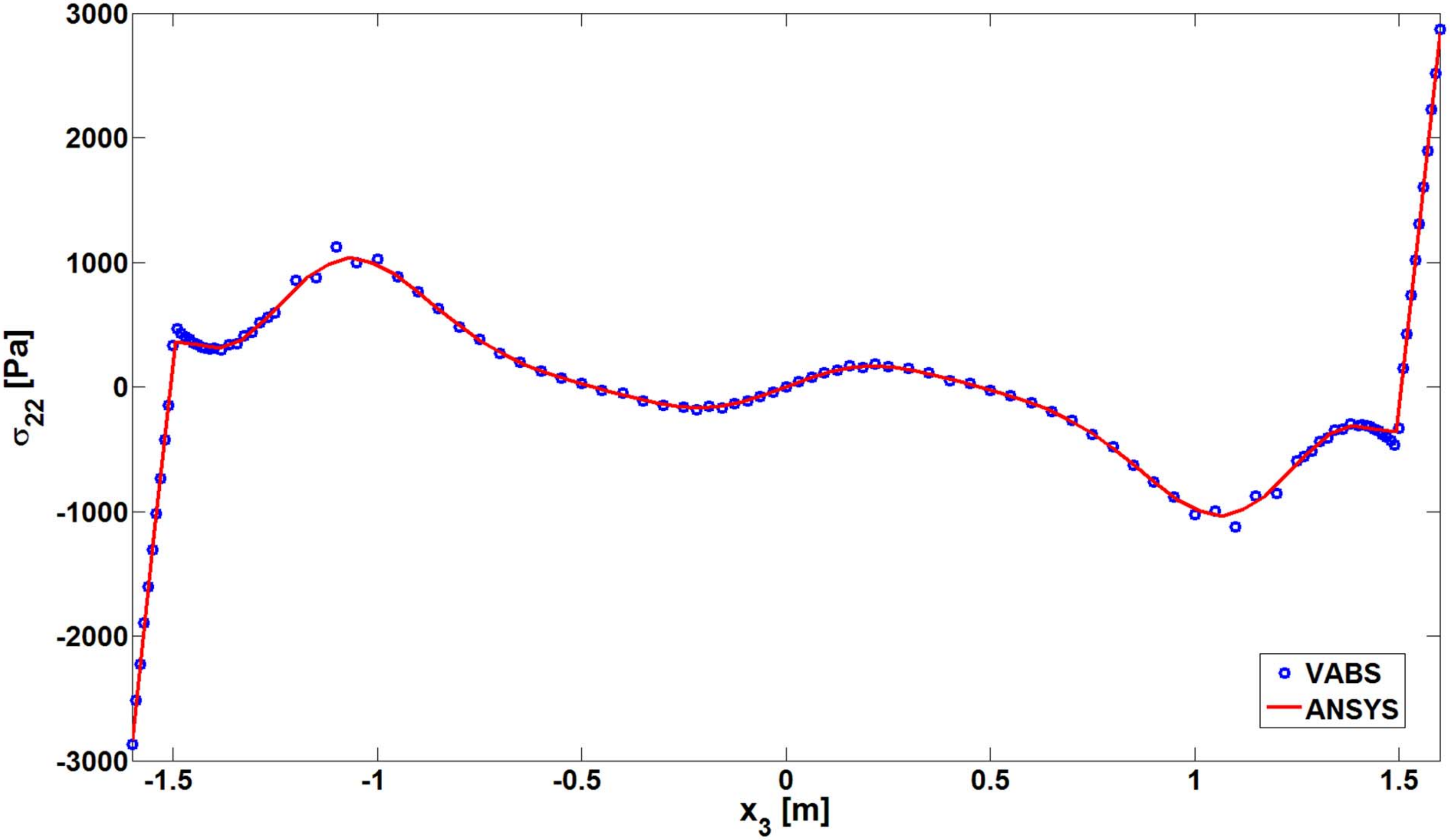
Heterogeneous Beam (cont.)



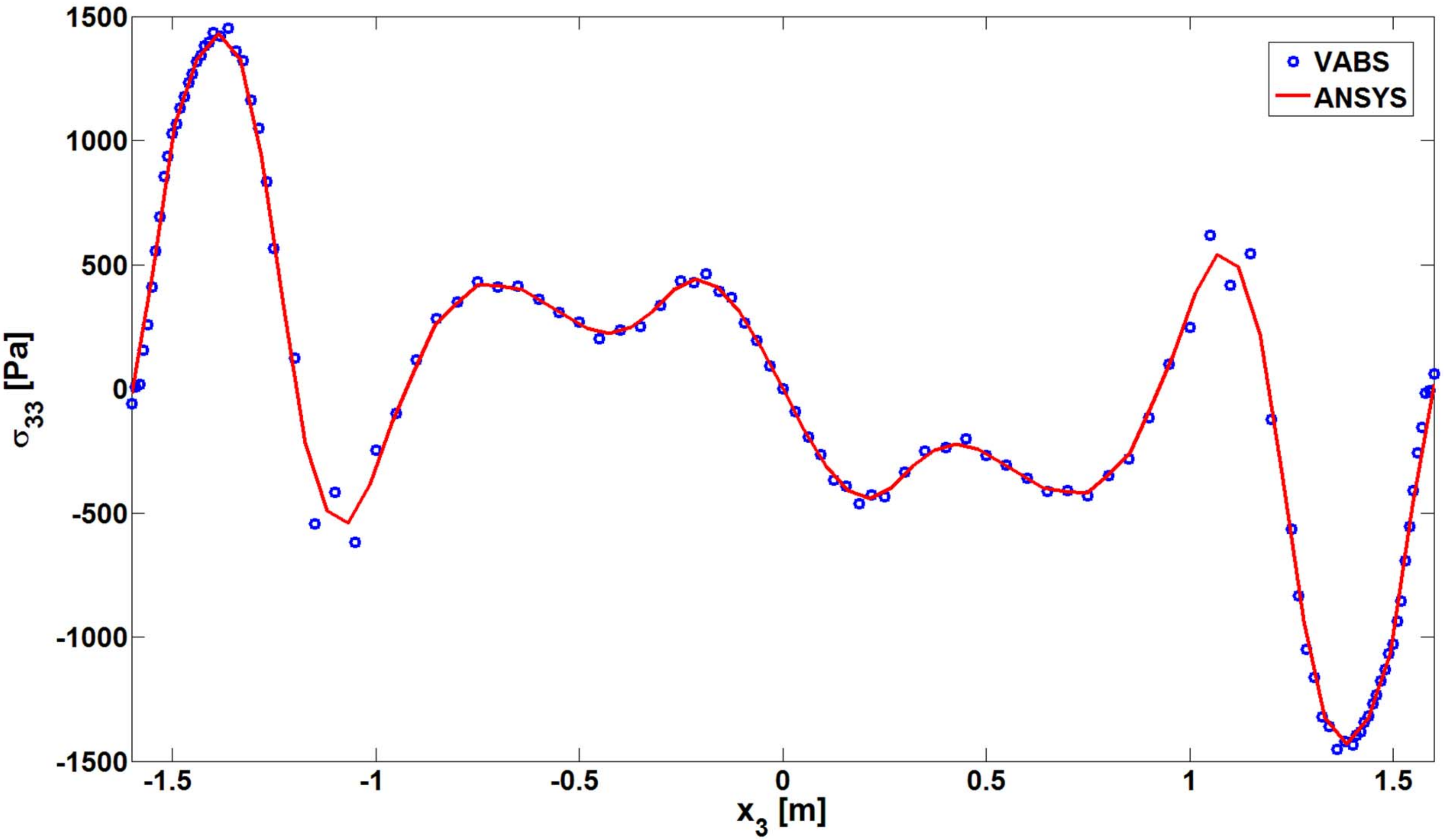
Heterogeneous Beam (cont.)



Heterogeneous Beam (cont.)



Heterogeneous Beam (cont.)



Current VABS R&D

- Model aperiodic spanwise heterogeneity: tapering (US Army)
- Model material nonlinearity: blade damping (US Army)
- Model geometrical nonlinearity: skin buckling (US Army)
- Model damaged blades (Army VLRCOE)
- More versatile preprocessor than PreVABS (work in progress with Utah Technology Commercialization & Innovation Program)
- Sensitivity analysis of VABS using more efficient methods than VABS-AD
- .....

Takeaway Messages

- VABS enables efficient high-fidelity analysis of composite blades using simple beam theories: best accuracy within given efficiency
  - Complete set of multiphysical properties: needed for static/dynamic analysis using beam elements
  - Complete set of multiphysical 3D fields (stress/strain)
- VABS code:
  - Highly optimized for efficiency: ply-level details of real blades can be modeled in seconds
  - Extensively validated in helicopter and wind industry
  - Directly integrated into other design environments
- 1D Beam analysis should accept full stiffness matrix to reap the full benefits of VABS
- More innovative VABS uses should be explored