WDRT: A toolbox for design-response analysis of wave energy converters

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ABSTRACT

In this paper, we present a numerical toolbox for design-response analysis of wave energy converters (WECs). The “WEC Design Response Toolbox (WDRT)” was developed during a series of efforts to better understand and improved the WEC survival design process. The WDRT has been designed as a tool for researchers and developers, enabling the straightforward application of statistical and engineering methods needed for design response analysis of a WEC, including characterization of environmental extremes, extreme response statistics, fatigue analysis and design wave composition. This paper gives a brief overview of the WDRT including its capabilities and underlying theory.

1. INTRODUCTION

The economic feasibility of wave energy converters (WECs) is dependent on both efficient energy absorption and efficient structural design. While WEC designers can draw from experiences in ship and offshore platform design, the unique characteristics of WECs, specifically their need to have large motions in order to perform their mission of energy capture, necessitates some further consideration. Uncertainty and lack of confidence in predictions for design responses/loads for WECs has contributed to the need for larger factors of safety in structural design and thus limited structural efficiency [1, 2]. In the aim of studying and improving the WEC design response analysis process, Sandia National Labs (SNL) and the National Renewable Energy Laboratory (NREL) have developed a set of tools to support the various sub-processes that make up the larger analysis process. These tools have been packaged into the WEC Design Response Toolbox (WDRT) for public usage at http://wec-sim.github.io/WDRT/.

2. TOOLBOX CAPABILITIES

The capabilities of the WDRT cover a range of components needed to perform design response analysis for a WEC:

- **Environmental Characterization** - characterization of extreme waves and development of contour lines based on empirical data
- **Short-term Extreme Response** - response statistics characterizing the extreme response/load for given sea state
- **Long-term Extreme Response** - response statistics characterizing the extreme response/load over an entire deployment lifetime
- **Fatigue** - quantification of an equivalent static load to account for fatigue loading
- **Design Wave Composition** - creation of “design waves” to represent entire sea states in limited time lengths for usage in expensive modeling techniques (e.g. computational fluid dynamics)

2.1 Environmental Characterization

The WDRT includes a package for creating environmental contours of extreme sea states using the methodology developed by Eckert-Gallup et al. [3] with additional improvements for characterizing the joint probability distribution of sea state variables of interest. Environmental contours describing extreme sea states can be used for numerical or physical model simulations analyzing the design-response of WECs. These environmental
contours, characterized by combinations of significant wave height \(H_s\) and either energy period \(T_e\) or peak period \(T_p\), provide inputs that are associated with a certain reliability interval. These reliability levels are needed to drive both contour and full sea state style long-term extreme response analyses discussed later in this paper.

The estimation of environmental contours of extreme sea states is a problem that has been tackled in many different ways. The classical inverse first-order reliability method (I-FORM) presented by Winterstein et al. [4], which utilizes the Rosenblatt transformation [5], provides the basis for many such methodologies. The first step of the basic application of the I-FORM method is to generate probability distributions for the variables under analysis. The traditional treatment of this step includes fitting distributions and parameter models to significant wave height \(H_s\) and energy period \(T_e\) to capture their inherent interdependency [6]. The second step of the classical I-FORM approach includes an evaluation of these fitted distributions at specific points to generate an environmental contour. The probability level for a given return period of interest (e.g., 100 years) is used as the radius of a circular isoline in the standard normal space. This isoline defines a group of equiprobable points characterized by the combination of two variables with individually varying probabilities that can be used to evaluate the fitted distributions of each variable of interest. The environmental contour for the given variables is the final result of this distribution evaluation.

The work of Eckert-Gallup et al. [3] presents a modified version of the I-FORM using principal component analysis (PCA) and improved joint probability distributions to improve final extreme sea state contours. This methodology is included in the WDRT. An example of an environmental contour created using this methodology is shown in Figure 1. It is important that a method for deriving this distribution be both rigorous and generally applicable to sites with varying physical properties and locations so that the contour methodology can maintain global robustness. Further refinement of the joint probability distribution of significant wave height and energy period described above utilizing Kernel Density Estimation (KDE) is under development for inclusion in a future version of the WDRT.

### 2.2 Short-term Extreme Response

The short-term extreme response module of the WDRT is a collection of functions useful for doing short-term extreme analysis on a response quantity of interest. Short-term in this context means sea-state specific and for a specific short-term period. For instance, you might be interested in 3-hour extreme mooring load on a floating device at a specific sea-state. In this case, 3 hours is the short-term period, and the mooring load is the response quantity of interest. The 3-hour extreme mooring load does not have a set value; that is, every three hour observation will not result in the same extreme value. Instead, it is defined by a probability distribution, the short-term extreme distribution.

The general starting point for all of these methods is a time-series of a quantity of interest at the specific sea-state under consideration. This time series may be generated via numerical model or empirical testing. As in other marine applications, when analyzing response of a WEC, the global peaks (largest point between consecutive zero up-crossings) can be considered an independent random variable [7]. Using one or more time-series for the response of quantity of interest in a sea state, you can use a number of different methods to estimate the short-term extreme distribution.

The short-term extreme module currently includes four different methods for estimating the short-term extreme distribution: all-peaks Weibull, Weibull tail-fit, peaks over threshold, and block maxima. The major difference between these methods is the trade-off they make between amount of data used and the relevance of that data to the extreme response. These four specific methods are described in more detail in [8].

The short-term extreme module includes methods for loading and processing data, estimating the short-term extreme distribution, and creating goodness-of-fit plots. Figure 2 shows an example of a short term extreme distribution obtained from a peaks distribution, from [8]. Here, a probability distribution (green line) has been created for the peaks in the PCC force for a given sea state. Using the peaks distribution, the extreme distribution (red line) is then obtained.

### 2.3 Long-term Extreme Response

With the short-term extreme response characterization methods and the environmental characterization methods discussed in the previous sections, a long-term extreme response for a design can be obtained. The long-term extreme response represents the design response for some specific deployment location and time-span. For example, these approaches can be used to obtain the design load (e.g., PCC force) over the expected 25-year deployment period for a location near NDBC buoy 46022. Two major classes of approaches...
are implemented in the WDRT: the contour approach and the full sea state approach.

2.3.1 Contour approach

In the contour approach, simulations are run along the desired extreme wave contour (e.g., 25-year contour). The condition producing the largest response is used to define the extreme response distribution for the device via a short-term extreme process. To obtain a single design response value for this method, one should select a percentile from that extreme response distribution based on some prior knowledge of system behavior. Typical percentiles used for marine structures range from 75 to 99% [8, 10, 11].

2.3.2 Full sea-state approach

In the full sea state approach, simulations are run at a sampling of sea states within an envelop defined by the environmental characterization process. Based on the device response and relative occurrence likelihood for each sea state, an extreme distribution is constructed. This distribution gives a richer picture of the design response and can, for example, be used to study how the design response varies with return period.

Figure 3 shows an example of the long term extreme complementary cumulative probability distribution (CCDF), which can be obtained from the full sea-state approach. The solid line shows the CCDF of the 1-hour storm PCC force. With this data, the expected return level for a given period (or the expected return period for a given level) can be obtained; Figure 3 shows that given a 25-year period, the expected return level is 2.8 MN. Additionally, a bootstrap approach was used to obtain the 95% confidence bounds for this distribution, which are shown in Figure 3 by dashed lines. This interval indicates the level of uncertainty in the analysis; this could potentially be reduced by obtaining additional data (i.e., running more tests or simulations).

2.4 Equivalent Design Wave

The time history data for short-term response analysis can be obtained through simulations for a specific time period. The statistical variation can further be included using Monte-Carlo-type approach if needed. Although the rapid improvement of computing technology has made the use of mid- and high-fidelity numerical methods available for design applications, the required computational time is still generally too long to handle. To address this, the ship and offshore drilling industries often perform their mid- and high-fidelity simulations and experimental wave tank tests using an equivalent design wave profile (see, e.g., [12]). The equivalent design waves are used to represent the full sea state at which the extreme loads most likely occur. Regular waves with a statistically determined maximum wave height under given probability level can be used for analyzing the extreme condition. However, the approach does not account for the influence from the instantaneous body position and the series of random waves [13]. Several other methods have been used for naval architecture studies, which constructed ensemble of short focused wave profiles that identified the design extreme events of the floating body based on the RAOs [14, 15]. These methods account for the instantaneous body position and the random wave elevation when the extreme events occur, based on the response RAOs (e.g., body motion, mooring moment and mooring load) and wave statistics. They assume that nonlinear response phenomena for a floating object can be approximated by a linearized response, and that the nonlinear response is a perturbation from that linear solution.

A most-likely response (MLER) approach is included in the WDRT. An example is presented in Figure 4 where the method was to model a floating ellipsoid in extreme wave environment ($H_s = 9.0 \text{ m}, T_p = 16.0 \text{ s}$), which is on the 100-year contour as shown in Figure 1. The ellipsoid was allowed to heave and pitch around the body center of gravity. In this example, the wave profile that produces the largest likely response of the WEC device was generated using the RAO calculated from a linear radiation-and-diffraction method based numerical model for each of the degrees of freedom of interest. The RAO is then combined with the Bretschneider spectrum corresponding to each of the five extreme wave environments using the MLER method to produce the underlying wave profile that leads to the largest response in each degree of freedom. The resulting wave profile.
Figure 4: Pitch response from the MLER method.

is then used in nonlinear WEC-Sim and high-fidelity unsteady Reynolds-averaged Navier–Stokes (URANS) simulations to fully model the system nonlinear hydrodynamics. The application of the MLER method enables a simple and efficient approach to evaluate the nonlinear extreme response for WECs. Further analyses, such as coupling to a finite-element analysis method, will be useful for more detailed system structural and design load analyses.

More details on the simulations and the example case study were presented in [16]. The high-fidelity CFD has the capability of capturing nonlinear wave and floating body interaction, which can be essential for predicting the device design load under extreme sea states. The MLER method allows the device response to events at the edges of the design envelope to be evaluated using a short period of wave profile, thus reducing the need for long period of simulation; this, in-turn, limits computational time.

2.5 Fatigue

In addition to the extreme loads discussed previously, a WEC must also be structurally able to withstand fatigue loading for its design lifetime. Fatigue loads are time varying loads which cause cumulative damage to structural components and eventually lead to structural failure. Usually, a component’s fatigue life/strength is reported in terms of an \( S-N \) curve. The \( S-N \) curve, which is typically obtained empirically, gives the number of load cycles \( N \) to failure at constant load amplitude \( S \), as illustrated in Figure 5. Mathematically, the behavior in Figure 5 is given by: \( \log(N) = \log(K) - m \log(S) \). Here, \( S_{ult} \) is the ultimate strength; \( S_{end} \) is the endurance limit below which no failure occurs with constant amplitude loading; \( m \) is the slope of the \( S-N \) curve, and \( K \) is an empirical material constant determining the level of the \( S-N \) curve.

WEC loads, however, are typically highly variable and by no means constant amplitude. The most common method used to predict the cumulative damage of variable loading is the Palmgren-Miner rule. The Palmgren-Miner rule is based on the assumption that the cumulative damage of each load cycle is sequence independent, and thus the total damage equivalent load, \( S_N \), is obtained with a linear summation of the distributed load ranges [17].

\[
S_N = \sqrt{\sum S_i^m n_i / N}
\]  

Here, \( S_i \) is the load range for bin \( i \), and \( n_i \) is the number of cycles for load range bin \( i \). The discretized load distribution used in (1) is usually obtained via the rain-flow counting method.

The intended used of the fatigue module in the WDRT is as an early design stage WEC fatigue load estimator. Required inputs to the module are: 1) A force or stress history, which may be obtained either experimentally or via simulation. Pertinent loads may include, power-take-off loads, mooring loads, bending moments, etc. 2) The \( S-N \) curve slope, \( m \), which is likely not known precisely in the early stages of design, but as an initial estimate, the following ranges may be used: \( m \approx 3 - 4 \) for welded steel, \( m \approx 6 - 8 \) for cast iron, and \( m \approx 9 - 12 \) for composites [17]. 3) And, \( N \), the number of cycles expected for the WEC’s design lifetime, which is up to the user to decide (\( N \) is likely on the order 500 for 1 hour and \( 4 \times 10^6 \) for 1 year). Given the prerequisite inputs, the fatigue module performs a rain-flow count of the load history, using the rain-flow counting algorithm outlined in [18], and then calculates the equivalent fatigue load using the Palmgren-Miner rule (1).

3. CONCLUSION

A openly-available toolbox has been developed to assist in the design response analysis of WECs and facilitate further research on this topic. The WDRT contains tools for characterization of environmental extremes, extreme response statistics, fatigue analysis and design wave composition. The WDRT is slated for public release in mid-summer of 2016 and will be made available at [http://wec-sim.github.io/WDRT/](http://wec-sim.github.io/WDRT/).

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5. REFERENCES


