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BLADE SHAPE FOR A TROPOSKIEN TYPE OF VERTICAL-AXIS WIND TURBINE

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ABSTRACT

The equations derived to define a troposkien (the shape a completely flexible cable assumes when it is spun at a constant angular velocity about a vertical axis to which its two ends are attached) are described. The implications of the solutions on the design of a vertical-axis wind turbine are discussed for cases where gravity is neglected.

List of Symbols

a	Maximum vertical displacement of cable, see Figure 1.
A	Area
A_s	Swept area, see Equation (25).
b	Maximum horizontal displacement of cable, see Figure 1.
C	Centrifugal force on the section of cable between the point of maximum deflection and point P, see Equation (2).
$E(\phi;k)$	Elliptic integral of the second kind with parameter k.
$F(\phi;k)$	Elliptic integral of the first kind with parameter k.
g	Acceleration due to gravity.
G	Gravitational force on the section of cable between the point of maximum deflection and point P, see Equation (4).
k	Parameter in the elliptic integrals, defined by Equation (11).
P	Point on the cable under discussion, see Figure 1.
r	Radial coordinate, see Figure 1.
r'	Dimensionless radial coordinate, see Equation (A5).
s	Length of cable between point of maximum deflection and point P, see Figure 1.
S	Total length of cable.
t	Dimensionless coordinate, see Equation (A16).
T	Tension in the cable at point P, see Figure 1.
T_o	Tension in the cable at the point of maximum deflection, see Figure 1.
z	Vertical coordinate, see Figure 1.
z'	Dimensionless vertical coordinate, see Equation (A5).
β	Ratio of maximum horizontal to maximum vertical displacement, see Equation (13).
γ	Parameter in the catenary equation, see Equation (A34).
ζ	Dummy variable of integration.
θ	Slope of the cable at point P, see Figure 1.
σ	Mass per unit length of the cable.
ϕ	Parameter in the elliptic integrals, see Equation (A16).
ω	Rotational angular velocity.
Ω	Rotational parameter, see Equation (12).

BLADE SHAPE FOR A TROPOSKIEN TYPE OF VERTICAL-AXIS WIND TURBINE

Introduction

In 1931 the U. S. Patent Office issued a patent in the name of G. J. M. Darrieus[1] for a vertical-axis wind turbine. His patent statement indicates that each blade should "have a stream-line outline curved in the form of a skipping rope." In the early 1970's, the National Research Council of Canada[2,3] independently developed the same concept of a vertical-axis wind turbine; in this concept, a perfectly flexible blade, under the action of centrifugal forces, assumes the approximate shape of a catenary. Hence, the major stresses in a catenary-shaped blade are tensile, i.e., negligible bending stress. However, the usual definition of a catenary is the shape assumed by a perfectly flexible cable of uniform density and cross section hanging freely from two fixed points. The word troposkien (from the Greek: τροπος, turning and σχοινιον, rope) will be used to describe the shape assumed by a perfectly flexible cable of uniform density and cross section if its ends are attached to two points on a vertical axis and it is then spun at constant angular velocity about the vertical axis.

The purpose of this report is to develop the equations that describe a troposkien and to discuss the implications of the solution.

Analysis

Figure 1 presents the schematic of a perfectly flexible cable rotating at a constant angular velocity ω about a vertical z axis. The displacement of the cable from the z axis is indicated by r. The origin of the r, z coordinate system has been chosen on the z axis at a point corresponding to the maximum deflection of the cable from the z axis. T_o, which is the tension in the cable at its point of maximum horizontal displacement, is directed vertically downward. The forces acting on the length of the cable between the point of maximum horizontal displacement and the point P are the centrifugal force C, the gravitational force G, and the two tensions T and T_o.

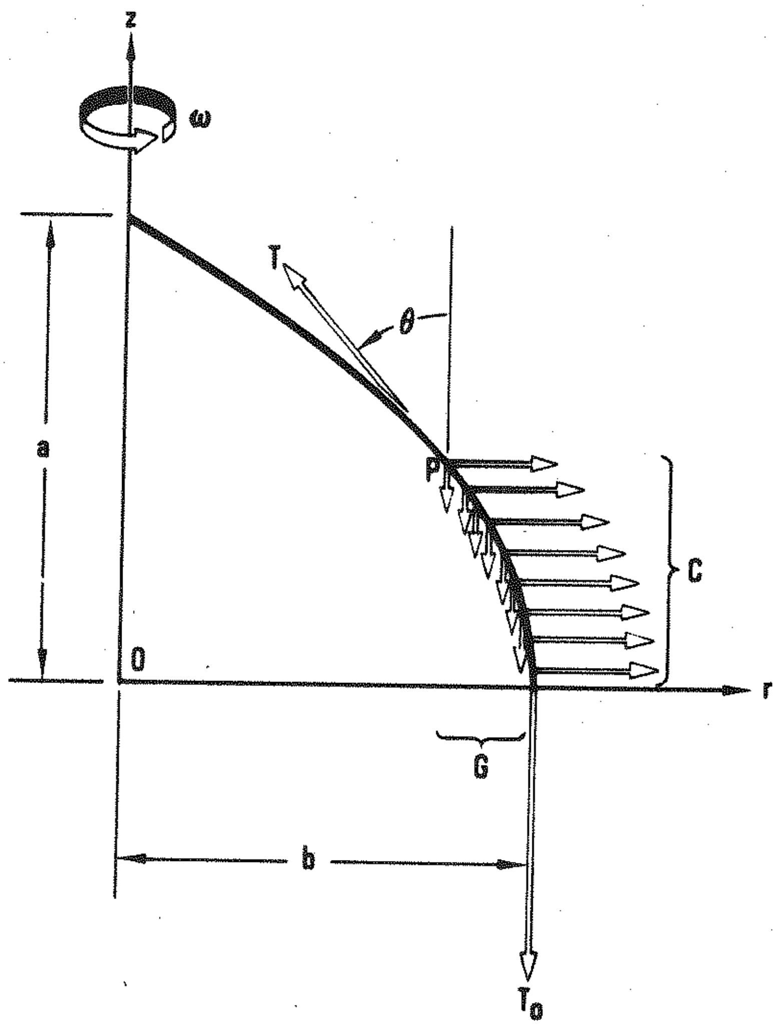

Figure 1. Schematic of a Perfectly Flexible Cable Rotating About a Vertical Axis

The balance of forces in the horizontal direction at point P requires that

$$T \sin \theta = C, \tag{1}$$

where the centrifugal force can be written as

$$C = \int_o^s \sigma \omega^2 r \, ds \ , \tag{2}$$

and where σ is the mass of the cable per unit length, and s is the length of the cable between the point of maximum horizontal deflection ($z = o$) and point P in Figure 1.

The balance of forces in the vertical direction at point P requires that

$$T \cos \theta = T_o + G \ , \tag{3}$$

where the gravitational force can be written as

$$G = \int_o^s \sigma g \, ds \tag{4}$$

and where g is the acceleration of gravity. If the mass per unit length is a constant, the gravitational force per unit length is constant, while the centrifugal force per unit length increases linearly with increasing distance from the axis of rotation. From the ratio of Equation (1) to Equation (3):

$$\tan \theta = \frac{C}{T_o + G} = - \frac{dr}{dz} \ . \tag{5}$$

Substituting Equations (2) and (4) into Equation (5) gives

$$\frac{dr}{dz} = - \frac{\sigma \omega^2 \int_o^s r \, ds}{T_o + \sigma g s} \ . \tag{6}$$

The boundary conditions to be met are

$$\left. \begin{aligned} r &= o \quad \text{at} \quad z = a \\ \frac{dr}{dz} &= o \quad \text{at} \quad z = o \ . \end{aligned} \right\} \tag{7}$$

At large rotational speeds, the cable tension T_o and $\sigma\omega^2$ will be large in comparison to σgs. If the gravitational forces are relatively small, σgs may be neglected, and the integro-differential equation describing the cable shape reduces to

$$\frac{dr}{dz} = -\frac{\sigma\omega^2}{T_o}\int_o^s r\,ds\,. \tag{8}$$

The solution to Equation (8), which is developed in the appendix, can be written as

$$\frac{z}{a} = \frac{\sqrt{1-k^2}}{\Omega}\left[F\left(\frac{\pi}{2};k\right) - F\left(\phi;k\right)\right] , \tag{9}$$

where $F(\phi;k)$ denotes the elliptic integral of the first kind with parameter k defined by

$$F(\phi;k) = \int_o^{\phi} \frac{d\zeta}{\sqrt{1-k^2\sin^2\zeta}} \quad . \tag{10}$$

In this expression, $\phi = \sin^{-1}(r/b)$ and b is the maximum horizontal displacement. Equation (10) cannot be expressed in terms of ordinary simple functions. Therefore, $F(\phi;k)$ must be taken from tables or evaluated numerically.

The parameter k is defined by

$$k^2 = \frac{1}{1+\left(\frac{2}{\Omega\beta}\right)^2} , \tag{11}$$

where

$$\Omega^2 = \frac{\sigma\omega^2a^2}{T_o} \tag{12}$$

and

$$\beta = \frac{b}{a} = \frac{2k}{\Omega\sqrt{1-k^2}} \quad . \tag{13}$$

The quantity Ω is a rotational parameter, and β is the ratio of the maximum horizontal displacement to the maximum vertical displacement. It should be noted that the angular velocity ω and the cable tension T_o always appear as the ratio ω^2/T_o.

Up to this point, the analysis has been expressed in terms of the three parameters k, β, and Ω. However, only two of these can be independent because of the interdependence shown by Equation (11).

If Equation (9) is evaluated at the point z = a, the following relationship between Ω and k results:

$$\Omega = \sqrt{1 - k^2}\ F\left(\frac{\pi}{2}; k\right) \quad . \tag{14}$$

On eliminating Ω from Equation (9) by substituting from Equation (14),

$$\frac{z}{a} = 1 - \frac{F(\phi; k)}{F\left(\frac{\pi}{2}; k\right)} \quad , \tag{15}$$

where, because the only parameter appearing explicitly on the right-hand side of Equation (15) is k, the solution can be reduced to a single family of curves with the parameter k. The elliptic integral subroutine ELLI developed by LASL[4] was used to evaluate Equation (15) numerically for several values of k. The results are shown in Figure 2. In that the parameter k has no obvious physical significance, the data presented in Figure 2 do not give much physical insight into the problem.

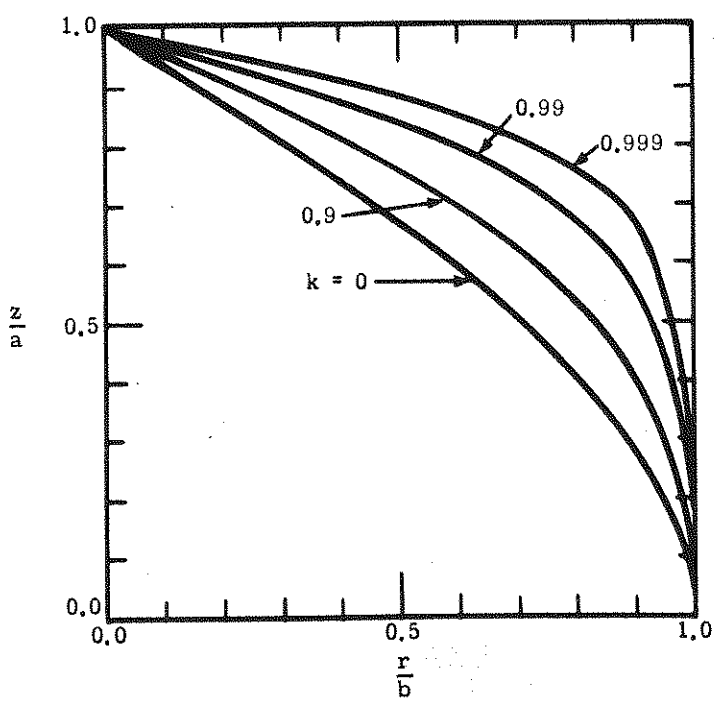

Figure 2. Nondimensional Cable Shape as a Function of the Parameter k

Suppose now that, instead of specifying the parameter k, a new parameter S, the total length of the cable, is specified. This S can be expressed as the integral

$$S = 2\int_o^a \sqrt{1 + \left(\frac{dr}{dz}\right)^2}\, dz \tag{16}$$

or, as shown in the appendix,

$$\frac{S}{2a} = \frac{2}{1 - k^2} \frac{E\left(\frac{\pi}{2}; k\right)}{F\left(\frac{\pi}{2}; k\right)} - 1 , \tag{17}$$

where $E(\phi; k)$ is the elliptic integral of the second kind with parameter k. Since the right-hand side of Equation (17) is a function of k only, a specification of the nondimensional cable length S/2a uniquely determines k. Equation (17) has been evaluated numerically, and the results are shown in Figure 3.

Once k has been determined from Equation (17) for a given S/2a, Ω can be computed from Equation (14) and β from Equation (13). Figures 4 and 5 show the variation of Ω and β with k, respectively. The shape of the cable as a function of the total cable length can now be determined by substituting these values of k, Ω, and β into Equation (9) and solving this equation for the relationship between z and r. Several calculated cable shapes, normalized by a, are shown in Figure 6. For comparison purposes, the catenary* shape is also shown. For a short length S/2a, the troposkien and catenary are very similar, but the difference increases with increasing cable length. It can be seen that, if the cable were to assume a semicircular shape, the nondimensional lengths S/2a would be $\pi/2$. Figure 7 compares the troposkien, catenary, and circle, each with $S/2a = \pi/2$. Both the troposkien and the catenary are different from the circular arc.

As is shown in the appendix, the cable tension ratio can be expressed as

$$\frac{T}{T_o} = 1 - \frac{\Omega^2 \beta^2}{2} \left(\frac{r^2}{b^2} - 1\right). \tag{18}$$

In this equation, the maximum value of T/T_o occurs at r = o. Therefore,

$$\left(\frac{T}{T_o}\right)_{max} = 1 + \frac{\Omega^2 \beta^2}{2} = 1 + \frac{2k^2}{1 - k^2} . \tag{19}$$

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*The equations for the catenary are given in the appendix, Equations (A33) through (A35).

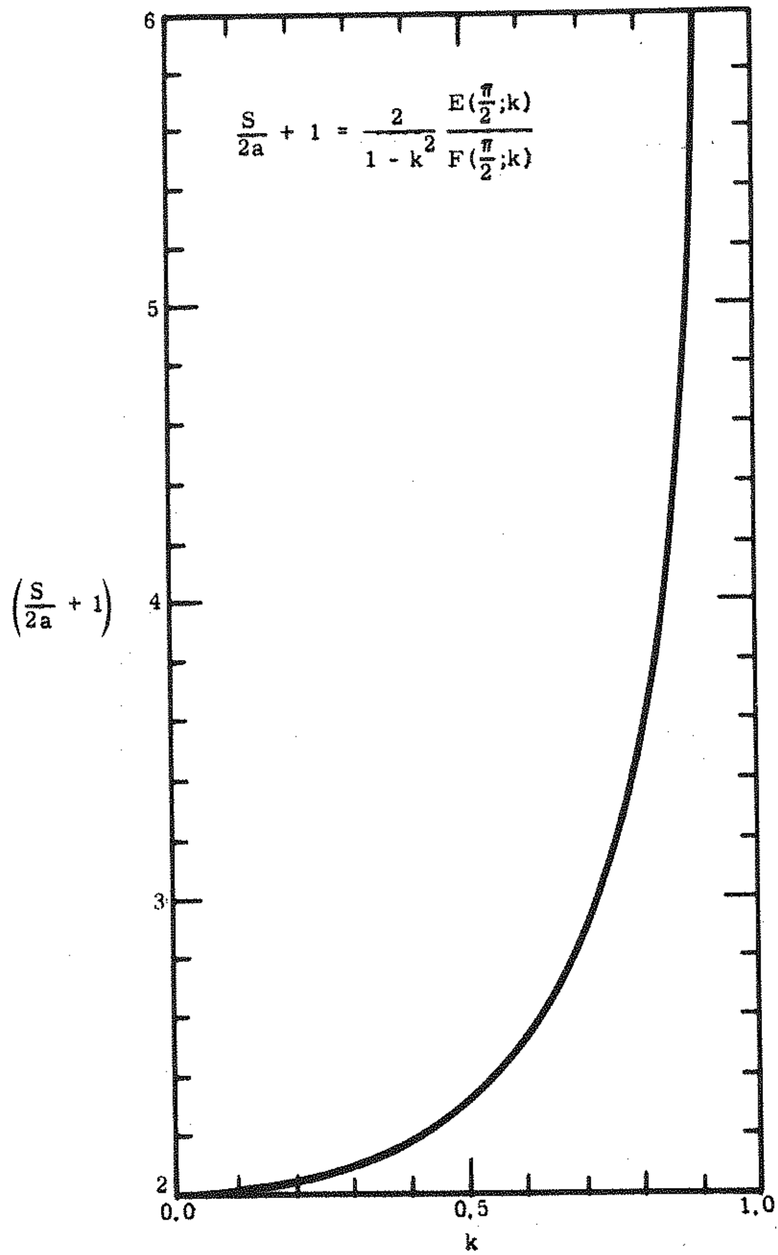

Figure 3. Nondimensional Cable Length as a Function of the Parameter k

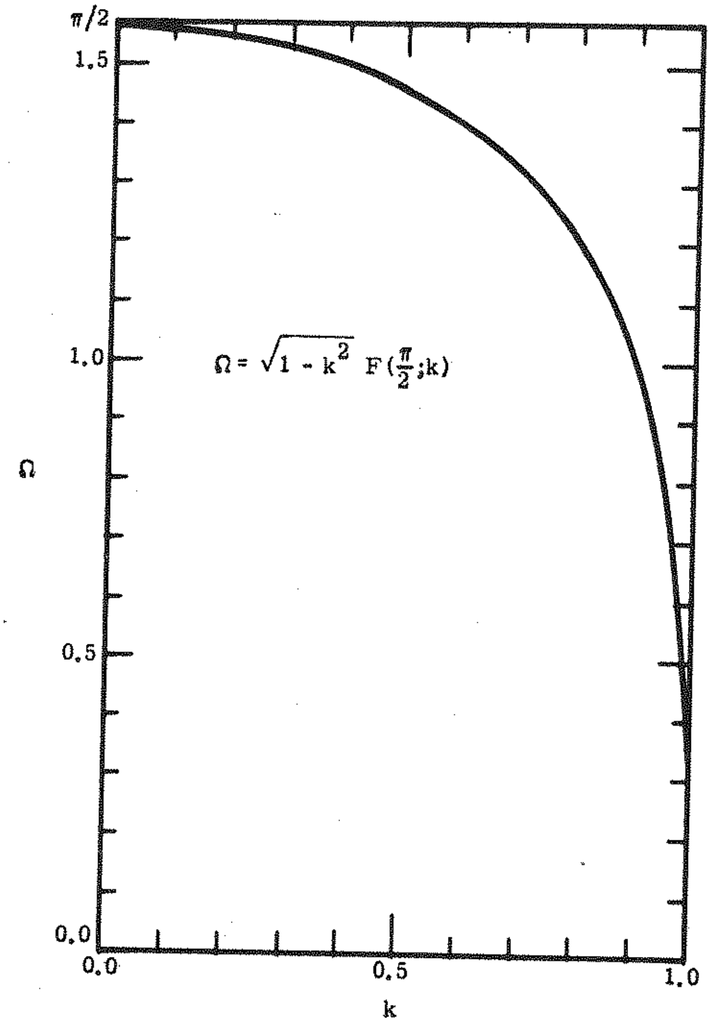

Figure 4. Variation of Rotational Parameter Ω with k

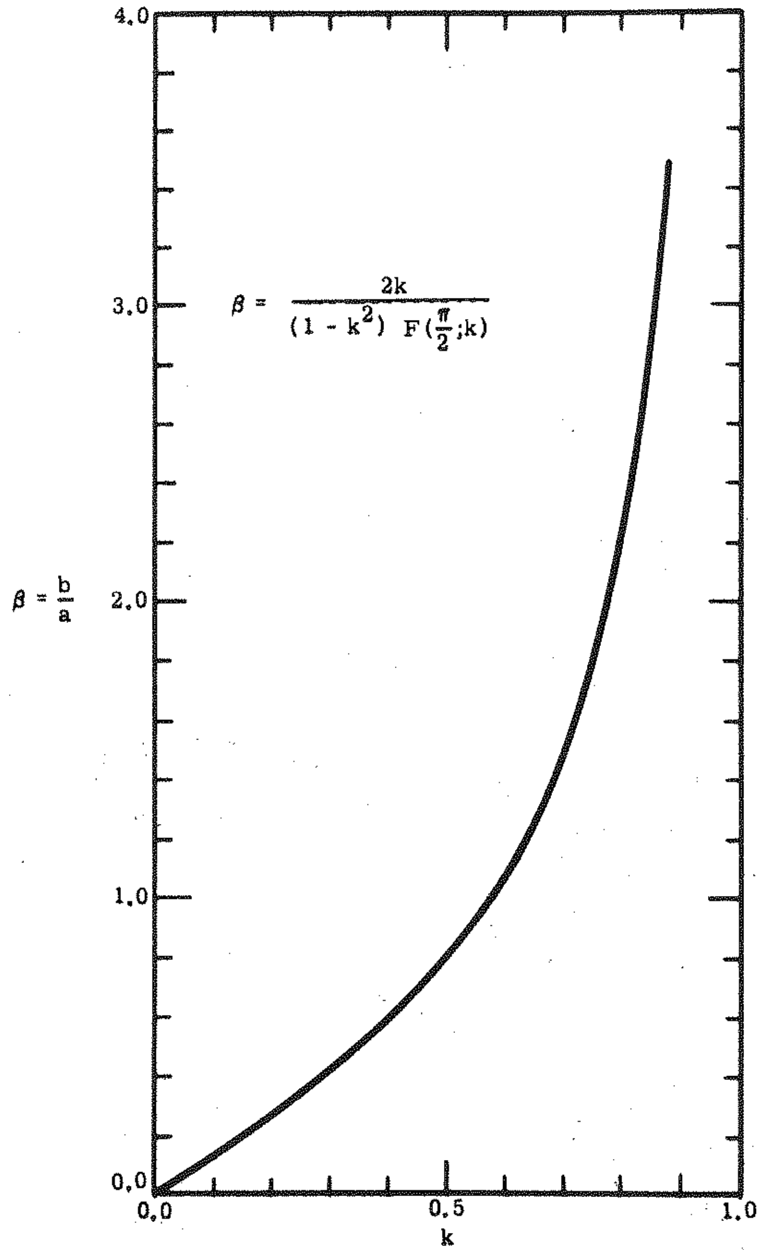

Figure 5. Variation of Maximum Displacement Parameter β with k

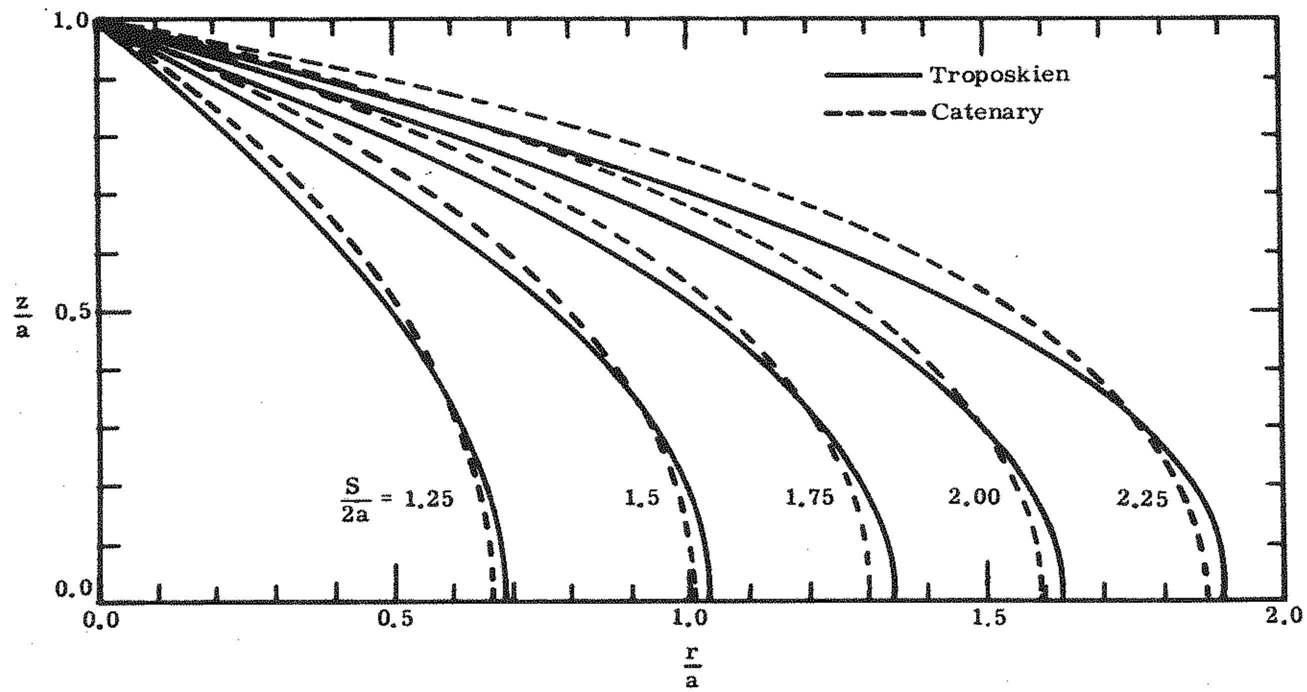

Figure 6. Dimensionless Cable Shape with Total Length as a Parameter

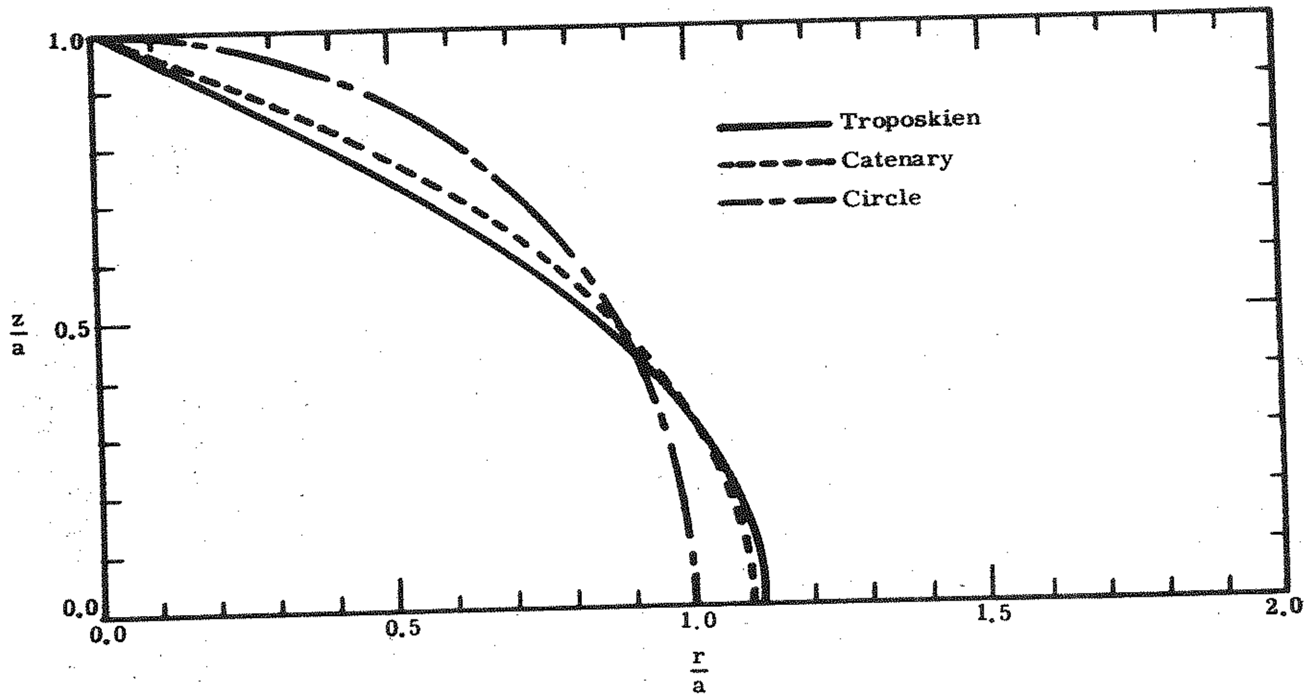

Figure 7. Comparison of Troposkien, Catenary, and Circle - All with $S/2a = \pi/2$

By using Equation (17) or the data in Figure 3, the cable tension ratio can be expressed as a function of the cable length. This relationship, which is very close to linear, is shown in Figure 8. Because, for a given cable length $S/2a$, the rotational parameter Ω is uniquely determined by Equations (17) and (14), a given $S/2a$ can be used to obtain the tension T_o at the point of maximum deflection by rearranging Equation (12):

$$T_o = \sigma a^2 \frac{\omega^2}{\Omega^2} \quad . \tag{20}$$

The maximum tension in the cable, which occurs at the point where the cable intersects the axis of rotation, can be determined by substituting the value of T_o obtained in Equation (20) into Equation (19), so that

$$T_{max} = T_o \left(1 + \frac{\Omega^2 \beta^2}{2}\right) \quad . \tag{21}$$

In dimensionless coordinates, the area under the troposkien curve is given by

$$\frac{A}{ab} = \int_o^1 \left(\frac{z}{a}\right) d\left(\frac{r}{b}\right) \quad . \tag{22}$$

From Equation (15),

$$\frac{A}{ab} = \int_o^1 \left[1 - \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)}\right] d\left(\frac{r}{b}\right) \tag{23}$$

Changing the integration variable from $\left(\frac{r}{b}\right)$ to ϕ, Equation (23) can be written as

$$\frac{A}{ab} = \int_o^{\pi/2} \cos\phi \left[1 - \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)}\right] d\phi \; . \tag{24}$$

The area under one quadrant of the troposkien is one-fourth of the area swept by the blades of a wind turbine; hence, the swept area can be computed by

$$\frac{A_s}{4ab} = \int_o^{\pi/2} \cos\phi \left[1 - \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)}\right] d\phi \quad . \tag{25}$$

Equation (25) was integrated numerically, and the results are shown in Figure 9.

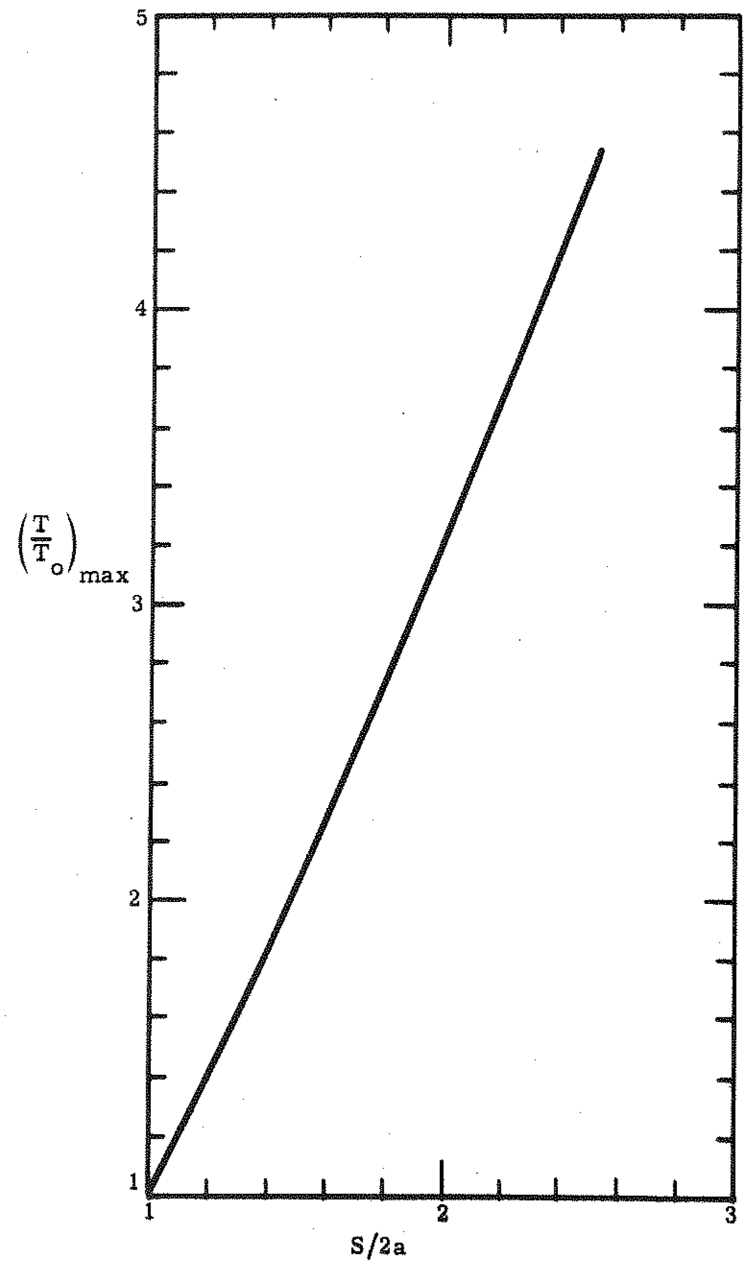

Figure 8. Tension Ratio as a Function of Cable Length

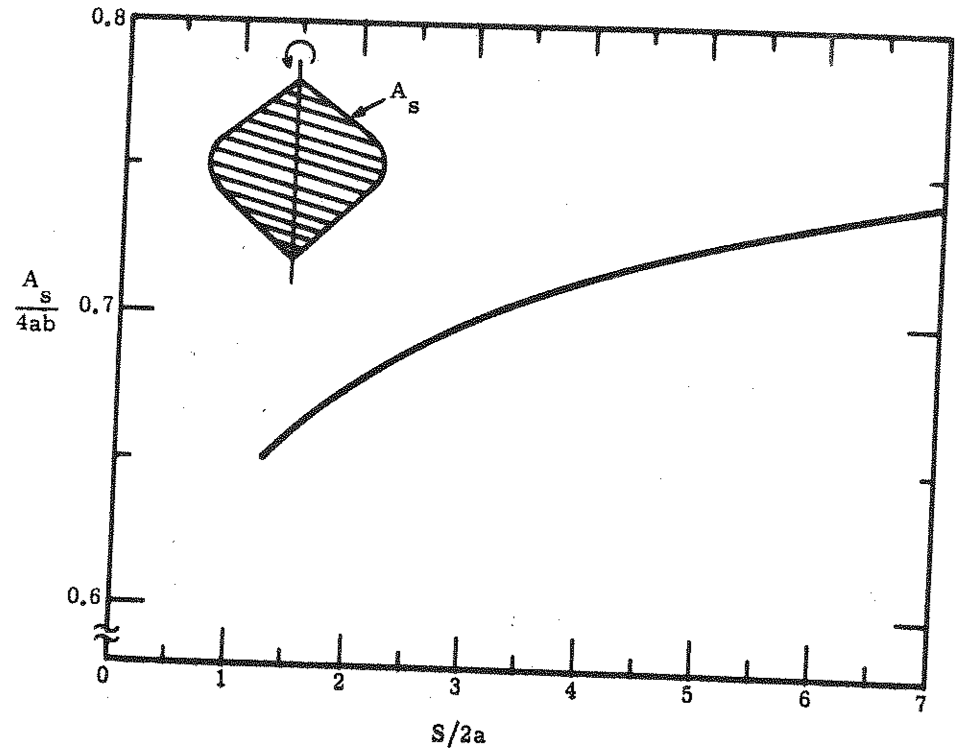

Figure 9. Dimensionless Swept Area as a Function of Dimensionless Cable Length

Observations

From the foregoing calculations several observations have been made.

For short cables (k small), the rotational parameter Ω approaches a maximum value of $\pi/2$. Figures 4 and 5 can be combined to give a plot of S/2a versus b/a as shown in Figure 10. This point, S/2a = 1, corresponds to the physically uninteresting case in which the cable lies on the axis of rotation.

The curves in Figure 6 also suggest that there may be an optimum blade length for a vertical-axis wind turbine. The driving torque on the blades will be the product of a chordwise force and a moment arm. The chordwise force is greatest on the vertical part of the blades, and the shorter blades have the greatest portion of the blade that is vertical. However, the shorter blades also have the smaller moment arm. Therefore, it is conceivable that based on force considerations, there is an optimum blade length. This problem is presently under investivation.

For a given cable length, which implies a given Ω, fixed pivot points, and a constant mass per unit length, the tension T_o varies directly as the square of the angular velocity ω. From Figure 10 it can be seen that increasing the cable length causes a decrease in the rotational parameter Ω which, for a given σ, a, and ω, in turn causes an increase in T_o according to Equation (20). Under these conditions, T_o increases with increasing length S/2a.

If the nondimensional cable length S/2a is specified, the cable shape is uniquely determined and is independent of the angular velocity ω. Also, the maximum cable tension occurs at the point where the cable is attached to the rotating shaft, and all tensions in the cable increase as the square of the angular velocity.

It should be emphasized that the above analysis and conclusions assume that gravitational forces are negligible. The case including gravity is being examined.

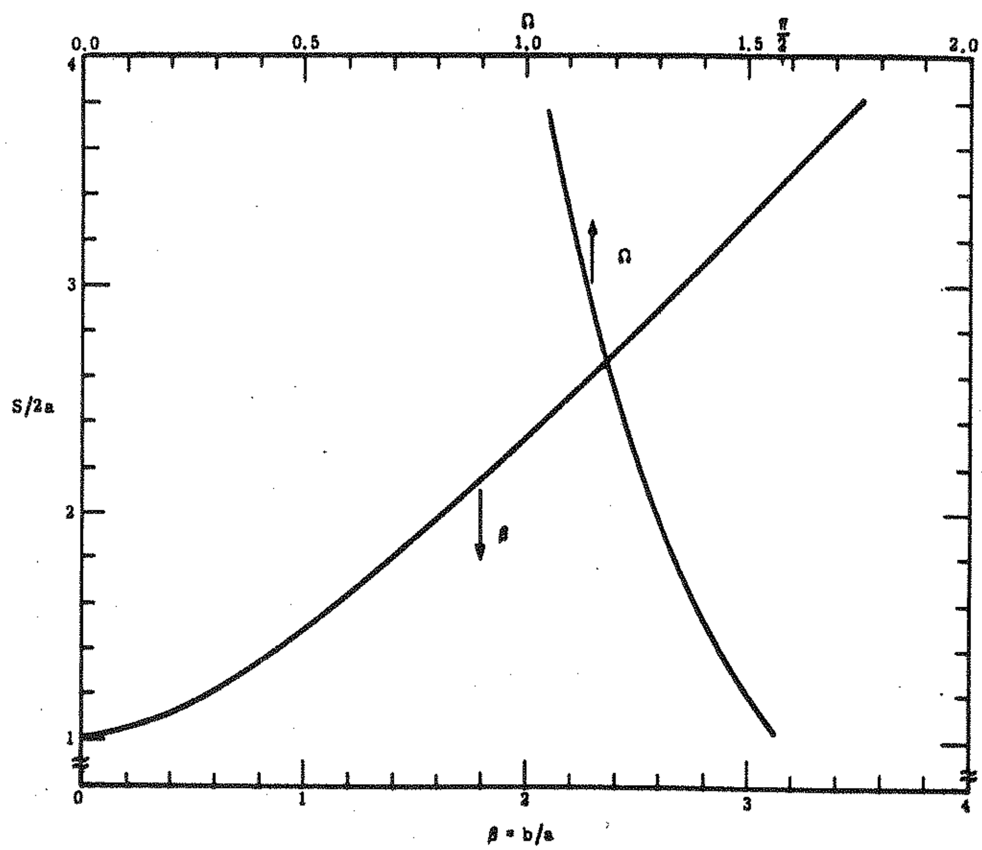

Figure 10. Variation of Cable Length with Maximum Displacement Parameter and Rotational Parameter

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3. P. South and R. S. Rangi, A Wind Tunnel Investigation of a 14 Ft. Diameter Vertical Axis Windmill, National Research Council of Canada, LTR-LA-105, September 1972.

4. Karl J. Melindez, Subroutine ELLI, LASL Programmers Information Manual, Vol. 4, Los Alamos Scientific Laboratory, May 1970.

APPENDIX

Referring to Figure 1, the integro-differential equation describing the cable shape if gravity is ignored is

$$\frac{dr}{dz} = -\frac{\sigma \omega^2}{T_o} \int_o^s r \, ds, \tag{A1}$$

subject to the boundary conditions

$$\left.\begin{aligned} \frac{dr}{dz} &= o \quad \text{at} \quad z = o, \\ r &= o \quad \text{at} \quad z = a. \end{aligned}\right\} \tag{A2}$$

The differential arc length ds is related to the cable slope by

$$ds = \sqrt{1 + \left(\frac{dr}{dz}\right)^2} \, dz . \tag{A3}$$

On substituting ds from Equation (A3) into Equation (A1),

$$\frac{dr}{dz} = -\frac{\sigma \omega^2}{T_o} \int_o^z r \sqrt{1 + \left(\frac{dr}{dz}\right)^2} \, dz . \tag{A4}$$

It is convenient to normalize all lengths by a, To do this, let

$$r' = \frac{r}{a}, \quad z' = \frac{z}{a}. \tag{A5}$$

From these definitions, Equation (A4) can be written as

$$\frac{dr'}{dz'} = -\Omega^2 \int_o^{z'} r' \sqrt{1 + \left(\frac{dr'}{dz'}\right)^2} \, dz' , \tag{A6}$$

where

$$\Omega^2 = \frac{\sigma \omega^2 a^2}{T_o} . \tag{A7}$$

In order to simplify the notation in what follows, the primes on r' and z' will be dropped with the understanding that r and z have both been normalized by a. In this notation, Equation (A6) and the boundary conditions, Equation (A2), are

$$\left.\begin{aligned} \frac{dr}{dz} &= -\Omega^2 \int_o^z r \sqrt{1 + \left(\frac{dr}{dz}\right)^2}\, dz \\ \frac{dr}{dz} &= o \quad \text{at} \quad z = o \\ r &= o \quad \text{at} \quad z = 1 \, . \end{aligned}\right\} \qquad (A8)$$

The integro-differential equation in Equation (A8) can be converted to a differential equation by differentiation:

$$\frac{d^2r}{dz^2} = -\Omega^2 r \sqrt{1 + \left(\frac{dr}{dz}\right)^2} \, . \qquad (A9)$$

This equation can be rearranged so that both sides of the equation are exact differentials:

$$\frac{d}{dz}\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2} = -\frac{\Omega^2}{2}\frac{d}{dz}\left(r^2\right) . \qquad (A10)$$

Integrating Equation (A10) and applying the boundary conditions at $z = o$, and setting $\beta = \frac{b}{a}$,

$$\frac{ds}{dz} = \sqrt{1 + \left(\frac{dr}{dz}\right)^2} = 1 - \frac{\Omega^2}{2}\left(r^2 - \beta^2\right) . \qquad (A11)$$

Solving this equation for the appropriate $\left(\frac{dr}{dz}\right)$,

$$\frac{dr}{dz} = -\frac{\Omega^2\beta^2}{2}\sqrt{1 + \frac{4}{\Omega^2\beta^2}}\left[\left(\frac{r^2}{\beta^2} - 1\right)\left(\frac{r^2}{\beta^2\left(1 + \frac{4}{\Omega^2\beta^2}\right)} - 1\right)\right]^{1/2} . \qquad (A12)$$

In order to put this equation into a form which can be solved in terms of standard functions, let a new parameter k and a new variable t be defined so that

$$t = \frac{r}{\beta} \; ; \; k^2 = \frac{1}{\left(1 + \frac{4}{\Omega^2\beta^2}\right)} \, . \qquad (A13)$$

Then Equation (A12) transforms to

$$\frac{dt}{dz} = - \frac{\Omega}{\sqrt{1 - k^2}} \left[\left(1 - t^2\right)\left(1 - k^2 t^2\right)\right]^{1/2} . \qquad (A14)$$

Integrating Equation (A14),

$$\frac{\Omega}{\sqrt{1 - k^2}} \int_o^z dz = -\int_1^t \frac{dt}{\left[\left(1 - t^2\right)\left(1 - k^2 t^2\right)\right]^{1/2}} . \qquad (A15)$$

The right-hand side of this equation, which is one form of an elliptic integral, can be converted to another form which is more convenient here by letting

$$t = \sin \phi = \frac{r}{\beta} . \qquad (A16)$$

The solution to this transformed equation is

$$z = \frac{1}{\Omega} \sqrt{1 - k^2} \left[F\left(\frac{\pi}{2}; k\right) - F\left(\phi; k\right)\right] , \qquad (A17)$$

where $F(\phi; k)$ is the elliptical integral of the first kind defined by

$$F(\phi; k) = \int_o^\phi \frac{d\zeta}{\sqrt{1 - k^2 \sin^2 \zeta}} . \qquad (A18)$$

Up to this point, the boundary condition at $z = 1$ has not been used. At this point, $\phi = 0$; therefore, Equation (A17) reduces to the constraint equation,

$$\Omega = \sqrt{1 - k^2}\; F\left(\frac{\pi}{2}; k\right) . \qquad (A19)$$

The normalized total cable length, $S/2a$, is given by

$$\frac{S}{2a} = \int_o^1 \sqrt{1 + \left(\frac{dr}{dz}\right)^2}\; dz . \qquad (A20)$$

Substituting from Equation (A11) into Equation (A20) gives

$$\frac{S}{2a} = \int_o^1 \left[1 - \frac{\Omega^2 \beta^2}{2} \left(\frac{r^2}{\beta^2} - 1\right)\right] dz . \qquad (A21)$$

The integration of the constant terms in Equation (A21) is straightforward; however, the integration of the r^2 term takes a little manipulation. This is done by first replacing r/β by $\sin \phi$ from Equation (A16). The integration variable is then changed from z to ϕ, where $d\phi$ is obtained by differentiating Equation (A17) and eliminating Ω by Equation (A19). The result of these operations is

$$\int_0^1 \frac{r^2}{\beta^2}\, dz = -\int_{\pi/2}^{0} \frac{\sin^2\phi\, d\phi}{F\left(\frac{\pi}{2};k\right)\sqrt{1 - k^2\sin^2\phi}}$$

$$= \frac{1}{k^2}\left[1 - \frac{E\left(\frac{\pi}{2};k\right)}{F\left(\frac{\pi}{2};k\right)}\right] , \tag{A22}$$

where $E(\phi;k)$ is the elliptic integral of the second kind defined by

$$E(\phi;k) = \int_0^{\phi} \sqrt{1 - k^2\sin^2\zeta}\, d\zeta . \tag{A23}$$

Substituting Equations (A22) and (A13) into Equation (A21) yields

$$\frac{S}{2a} = \frac{2}{1 - k^2}\,\frac{E\left(\frac{\pi}{2};k\right)}{F\left(\frac{\pi}{2};k\right)} - 1 . \tag{A24}$$

If $S/2a$ is specified, then Equation (A24) can be iteratively solved for the parameter k. This has been done for several values of $S/2a$ where Newton's method was found to work well in determining k. The procedure is as follows.

Define a function $G(k)$ from Equation (A24) so that

$$G(k) = E\left(\frac{\pi}{2};k\right) - \frac{1}{2}\left(\frac{S}{2a} + 1\right)\left(1 - k^2\right) F\left(\frac{\pi}{2};k\right) \tag{A25}$$

and search for the value of k that forces $G(k)$ to zero. The iteration sequence is

$$k_{n+1} = k_n - \frac{G\left(k_n\right)}{\frac{d}{dk}\left[G\left(k_n\right)\right]} , \tag{A26}$$

where

$$\frac{dG(k)}{dk} = \frac{dE\left(\frac{\pi}{2};k\right)}{dk} + \left(\frac{S}{2a} + 1\right) kF\left(\frac{\pi}{2};k\right)$$

$$- \frac{1}{2}\left(\frac{S}{2a} + 1\right)\left(1 - k^2\right)\frac{dF\left(\frac{\pi}{2};k\right)}{dk} , \tag{A27}$$

$$\frac{dF\left(\frac{\pi}{2};k\right)}{dk} = -\frac{1}{k}\left[F\left(\frac{\pi}{2};k\right) - \frac{1}{1-k^2}\,E\left(\frac{\pi}{2};k\right)\right] , \tag{A28}$$

$$\frac{dE\left(\frac{\pi}{2};k\right)}{dk} = \frac{1}{k}\left[E\left(\frac{\pi}{2};k\right) - F\left(\frac{\pi}{2};k\right)\right] . \tag{A29}$$

In the calculation reported in this paper, the elliptic integrals were evaluated by means of the ELLI program obtained from LASL.[4] The iteration sequence given by Equation (A26) generally converged in 6 to 8 iterations. The time required for the evaluation of $E(\phi;k)$ or $F(\phi;k)$ on the CDC 6600 was approximately 0.7 millisecond.

The one variable in the problem which has not yet been determined by the analysis is the tension T in the cable. This is easily obtained by noting in the body of the paper that, if Equations (1) and (3) are squared and added,

$$T^2 = (T_o + G)^2 . \tag{A30}$$

If the gravitational forces are neglected and dr/dz is substituted from Equation (5),

$$\frac{T}{T_o} = \sqrt{1 + \left(\frac{dr}{dz}\right)^2} . \tag{A31}$$

This can be put in terms of the parameters of the problem by substituting from Equation (A11) into Equation (A31):

$$\frac{T}{T_o} = 1 - \frac{\Omega^2\beta^2}{2}\left(\frac{r^2}{\beta^2} - 1\right) . \tag{A32}$$

For the sake of completeness, the equation for the catenary is also presented. From any standard mechanics text,

$$r = \frac{1}{\gamma}\,(\cosh\gamma - \cosh\gamma z) , \tag{A33}$$

where

$$\gamma = \frac{\sigma a}{T_o} \tag{A34}$$

It should be noted that both r and z have been normalized by a. The length constraint is given by

$$\frac{S}{2a} = \frac{\sinh\gamma}{\gamma} \tag{A35}$$

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