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Some Geometrical Aspects of Troposkiens as Applied to Vertical-Axis Wind Turbines

B. F. Blackwell, G. E. Reis

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SOME GEOMETRICAL ASPECTS OF TROPOSKIENS AS APPLIED TO VERTICAL-AXIS WIND TURBINES

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ABSTRACT

Several geometric ratios connected with the troposkien (the shape a completely flexible cable assumes when it is spun at a constant angular velocity about a vertical axis to which its two ends are attached) are derived analytically. The results are evaluated for several values of the ratio of the maximum radial displacement to half the spacing between the attachment points. The results are given in the form of equations, tables, and plots. A comparison is made with a particular parabola which approximated the troposkien.

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SYMBOLS

a	Distance from origin to the intercept of the troposkien and the z axis
A	Area
A_c	Blade cross-sectional area
A_s	Frontal area swept by a troposkien type turbine
b	Distance from the origin to the point of maximum radial displacement
C_{T_1}, C_{T_2}	Dimensionless tensile stress coefficients
C_Q	Single blade torque coefficient
d	Separation between troposkien and parabola measured normal to the troposkien
$E(\phi, k)$	Elliptic integral of the second kind
$F(\phi, k)$	Elliptic integral of the first kind
G	Convergence parameter
I_z	Single blade mass moment of inertia about the z axis
k	Dimensionless tension parameter
N	Number of blades in a wind turbine
Q_L	Torque exerted on the turbine by the load
r	Radial coordinate
$\bar{r}$	Radial coordinate of the centroid of the area under the troposkien in the first and fourth quadrants
s	Arc length measured from the r axis to a point on the troposkien or parabola
S	Blade length measured between the points of attachment on the axis of rotation
t	Time
T	Tension in the blade
T_o	Tension in the blade at point of maximum displacement
u, v	Dummy variables of integration
V	Wind velocity
z	Axial coordinate
β	Length ratio, $\beta = b/a$
δ	Blade mass per unit length
ρ	Uniform blade mass density
ρ_a	Density of air
ρ_e	Effective blade density
ϕ	Variable in the elliptic integrals, $\phi = \sin^{-1}\left(\frac{r}{b}\right)$
ω	Angular velocity of the turbine about the z axis
Ω	Dimensionless rotational parameter

The subscript p is used to identify quantities associated with the parabola.

SUMMARY

A previous report[1] developed the troposkien shape and some of its geometrical characteristics in terms of a dimensionless cable length. For engineering purposes, a more useful parameter was found to be β, which is the ratio of the maximum radial displacement to half the distance between the attachment points on the axis of rotation.

In the present report, expressions are derived for normalized arc lengths, area ratios, stress coefficients, the tension ratio, the mass moment of inertia and the centroid. These expressions are then used to draw a series of conclusions with regard to a troposkien wind turbine.

It is shown that the ratio of the swept area to the square of length of a turbine blade has a maximum in the vicinity of $\beta = 1.0$. This peak was calculated numerically to occur at $\beta = 0.99458568$. Tensile stress coefficients are developed and it is shown from these that if the effective blade density remains constant then the maximum tensile stress is independent of size. An equation for the mass moment of inertia is derived and if normalized in a manner which follows directly from the equation of motion, the resulting nondimensional moment of inertia is shown to be expressible as a function of β only, indicating that the dynamic behavior of a large system and a small system are similar provided the blade linear densities are the same. On examining the results obtained in the analysis, it appears that a practical wind turbine with blades curved in the troposkien shape should have a β near unity.

A comparison is also made between a troposkien and an approximating parabola. The fit is sufficiently close that it appears that the parabola may be used as a practical engineering approximation to a troposkien for values of β between 0.8 and 1.7, at least for the prediction of aerodynamic performance.

The results of the analysis are presented in the form of equations, tables, and plots.

SOME GEOMETRICAL ASPECTS OF TROPOSKIENS AS APPLIED TO VERTICAL-AXIS WIND TURBINES

Introduction

In Reference 1 a troposkien is defined as the shape assumed by a perfectly flexible cable of uniform density and cross section when its ends are attached to two points on a vertical axis and then spun at a constant angular velocity about this axis. A schematic diagram of a troposkien and the coordinate system used in analyzing it are shown in Figure 1. This shape is significant in the design of the Darrieus[2] type of vertical-axis wind turbine since blades fabricated with this profile will be free of bending stresses when air loading and gravity are disregarded.

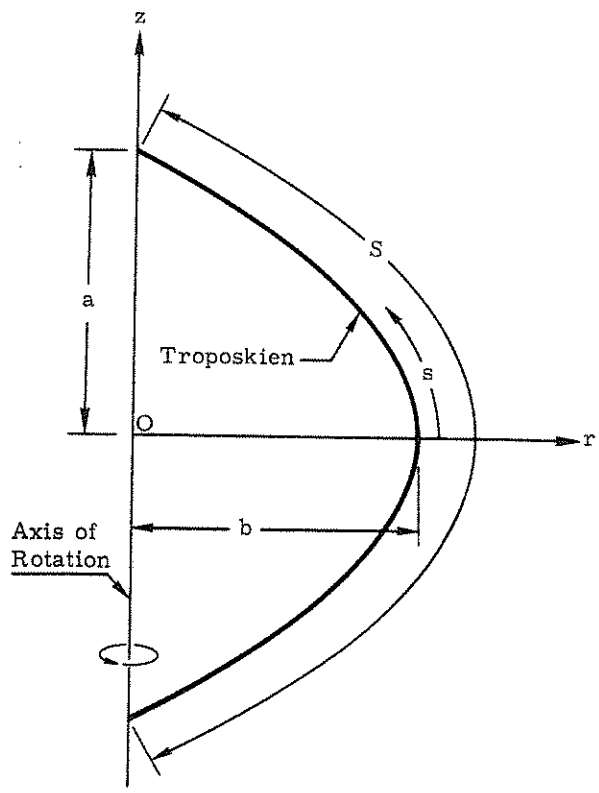

Figure 1. Coordinate system used in the analysis of the troposkien

The dimensionless troposkien shape (z/a versus r/a) is uniquely determined when any one of four naturally occurring parameters is specified. Table I lists these parameters and their physical significance. The analysis in Reference 1, which is used as the basis for the present work, used S/(2a) as the principal parameter with the other parameters being expressed as functions of S/(2a). Subsequent studies of the troposkien shape have indicated that the parameter $\beta = b/a$ is a more useful parameter.

TABLE I

LIST OF DIMENSIONLESS TROPOSKIEN PARAMETERS

Parameter	Definition	Physical Significance
$S/2a$		Normalized cable length
β	$= b/a$	Normalized maximum deflection
Ω	$= a\omega\left[\delta/T_o\right]^{1/2}$	Normalized rotation
k	$= \left[\dfrac{\left(\frac{T}{T_o}\right)_{max} - 1}{\left(\frac{T}{T_o}\right)_{max} + 1}\right]^{1/2}$	Tangent of 1/2 the angle between the axis of rotation and the troposkien at the point of attachment

The purpose of the present report, which is an extension of the analysis presented in Reference 1, is to examine the troposkien's shape and characteristics as a function of the parameter β. The actual shapes of troposkiens were calculated for several values of β and are compared with a set of parabolas which have been proposed[3] as an approximation to the troposkien. In addition the troposkien shape which has the maximum area for a given cable length between the points of attachment on the axis of rotation has been determined. Dimensionless tension coefficients and the moment of inertia are also derived. The results of these analyses are presented in the form of equations, tables and plots.

In the analysis that follows, it is assumed that the description of troposkien curve also refers to a description of a blade of a troposkien type of vertical-axis wind turbine and as a result the discussion at times moves back and forth between the mathematical description of the troposkien and the wind turbine.

Analysis

The equation describing the troposkien was derived in Reference 1 and is

$$\frac{z}{a} = 1 - \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)} \tag{1}$$

where

$$\phi = \sin^{-1}\left[\frac{1}{\beta}\left(\frac{r}{a}\right)\right] . \tag{2}$$

The relationship between the parameters k and ϕ was also derived in Reference 1 and is given by the equation

$$\beta = \frac{2k}{(1 - k^2)F\left(\frac{\pi}{2};k\right)} \quad . \tag{3}$$

Consequently Eq. (1) can be used to generate a plot of z/a versus r/a with β as a parameter by determining the necessary k in Eq. (1) for any given β by means of Eq. (3). This was done for selected numerical values of β between 0 and 2.5. The results are given in Appendix A. Plots of several of the cases listed in this table are shown as the solid lines in Figure 2.

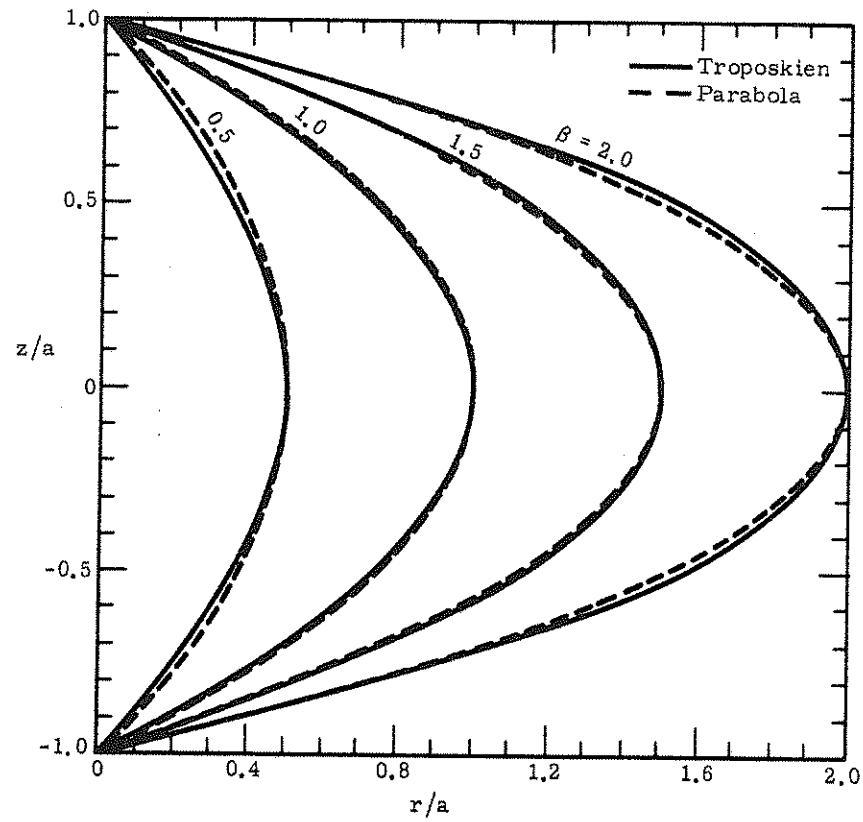

Figure 2. Plots of normalized troposkiens and the associated approximating parabolas for β's of 0.5, 1.0, 1.5 and 2.0

Since the equation which describes a troposkien contains elliptic integrals, which are somewhat inconvenient to work with, some investigators have proposed using simple algebraic expressions as approximations to the troposkien for design purposes. One such proposal[3] is the parabola

$$\left(\frac{z}{a}\right)^2 = 1 - \frac{1}{\beta}\left(\frac{r}{a}\right) \tag{4}$$

which agrees identically with the troposkien at the end points, z/a = 0 and z/a = 1. Eq. (4) has been solved for the same values of β as used for the troposkiens in Appendix A. The numerical results are listed in Appendix B. Plots of these parabolas which have the same β's as the ones used for the troposkiens in Figure 2 are shown as the dashed lines in this same figure. For the

β's used in Figure 2, the parabolas lie quite close to the corresponding troposkien. Values of the normalized separation, d/a and d/b, were calculated. These are listed in Table II and are plotted in Figure 3. As can be seen in the figure, both d/a and d/b are less than 2 percent for $0.8 \leq \beta \leq 1.7$, consequently, the parabola defined in Eq. (4) could probably be used as a mathematical model for predicting the aerodynamic performance of a troposkien type of vertical axis wind turbine without serious error for β's lying in this range. However, before attempting to manufacture a turbine blade with a parabolic profile, consideration should be given to the bending stresses due to this approximation. In fact, it would be as easy to manufacture a blade with a troposkien shape as one with a parabolic shape since both may require numerically controlled milling.

TABLE II

NORMALIZED MAXIMUM SEPARATIONS, d/a AND d/b, BETWEEN TROPOSKIENS AND THE APPROXIMATING PARABOLA FOR SELECTED VALUES OF β BETWEEN ZERO AND 2.5

β	d/a	d/b	Troposkien Coordinate z/a at Maximum Separation
0.0001	0.000005602	0.05602	0.7000
0.1	0.005492	0.05492	0.7004
0.2	0.01037	0.05185	0.7015
0.3	0.01417	0.04723	0.6934
0.4	0.01669	0.04173	0.6959
0.5	0.01793	0.03586	0.6987
0.6	0.01803	0.03005	0.7019
0.7	0.01722	0.02460	0.7052
0.8	0.01577	0.01971	0.7278
0.9	0.01390	0.01544	0.7499
1.0	0.01186	0.01186	0.7716
1.1	0.009815	0.008922	0.8019
1.2	0.007906	0.006588	0.8244
1.3	0.006199	0.004768	0.8512
1.4	0.009325	0.006660	0.4071
1.5	0.01278	0.008520	0.4322
1.6	0.01633	0.01021	0.4362
1.7	0.01991	0.01171	0.4506
1.8	0.02349	0.01305	0.4544
1.9	0.02704	0.01423	0.4685
2.0	0.03054	0.01527	0.4721
2.1	0.03398	0.01618	0.4756
2.2	0.03736	0.01698	0.4790
2.3	0.04066	0.01768	0.4823
2.4	0.04389	0.01829	0.4752
2.5	0.04705	0.01882	0.4783

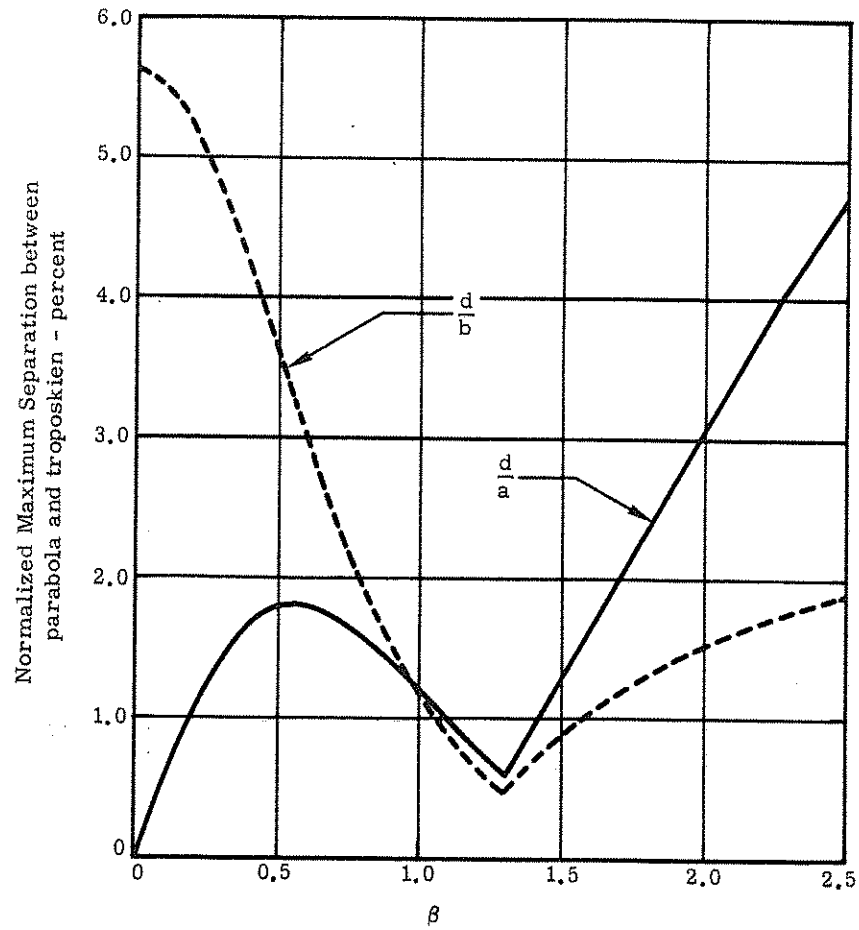

Figure 3. Plots of the normalized maximum separation between troposkiens and the associated approximating parabolas as a function of β for β between zero and 2.5

It has been found useful to relate the incremental arc length s along the troposkien to the axial coordinate z. This can be done by starting with Eq. (A11) in Reference 1, which is

$$\frac{d\left(\frac{s}{a}\right)}{d\left(\frac{z}{a}\right)} = -\frac{\Omega}{2}\left[\left(\frac{r}{a}\right)^2 - \beta^2\right] . \tag{5}$$

Integrating this yields

$$\frac{s}{a} = \frac{2}{(1 - k^2)F\left(\frac{\pi}{2};k\right)}\left[E\left(\frac{\pi}{2};k\right) - E\left(\phi;k\right)\right] + \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)} - 1. \tag{6}$$

The detailed development of Eq. (6) is presented in Appendix C.

The length of the troposkien between $z/a = 0$ and $z/a = 1$ can be determined by substituting $\phi = 0$, corresponding to $z/a = 1.0$, into Eq. (6). The total length of the troposkien between $z/a = -1$ and $z/a = +1$ is then given by

$$\frac{S}{2a} = \frac{2}{(1-k^2)} \frac{E\left(\frac{\pi}{2}; k\right)}{F\left(\frac{\pi}{2}; k\right)} - 1. \tag{7}$$

The incremental arc length normalized by the total cable length can be found by dividing Eq. (6) by Eq. (7),

$$\frac{s}{(S/2)} = \frac{\left\{\frac{2}{(1-k^2)\,F\left(\frac{\pi}{2}; k\right)}\left[E\left(\frac{\pi}{2}; k\right) - E(\phi;k)\right] + \frac{F(\phi; k}{F\left(\frac{\pi}{2}; k\right)} - 1\right\}}{\left[\frac{2}{(1-k^2)} \frac{E\left(\frac{\pi}{2}; k\right)}{F\left(\frac{\pi}{2}; k\right)} - 1\right]} \tag{8}$$

Eq. (8) has been evaluated for those values of β used previously. The numerical values are given in Appendix B and are plotted in Figure 4. In this plot it can be seen, as an example, that for $\beta = 1.0$, approximately 50 percent of the arc length lies inside the points where z is approximately 60 percent or more of a.

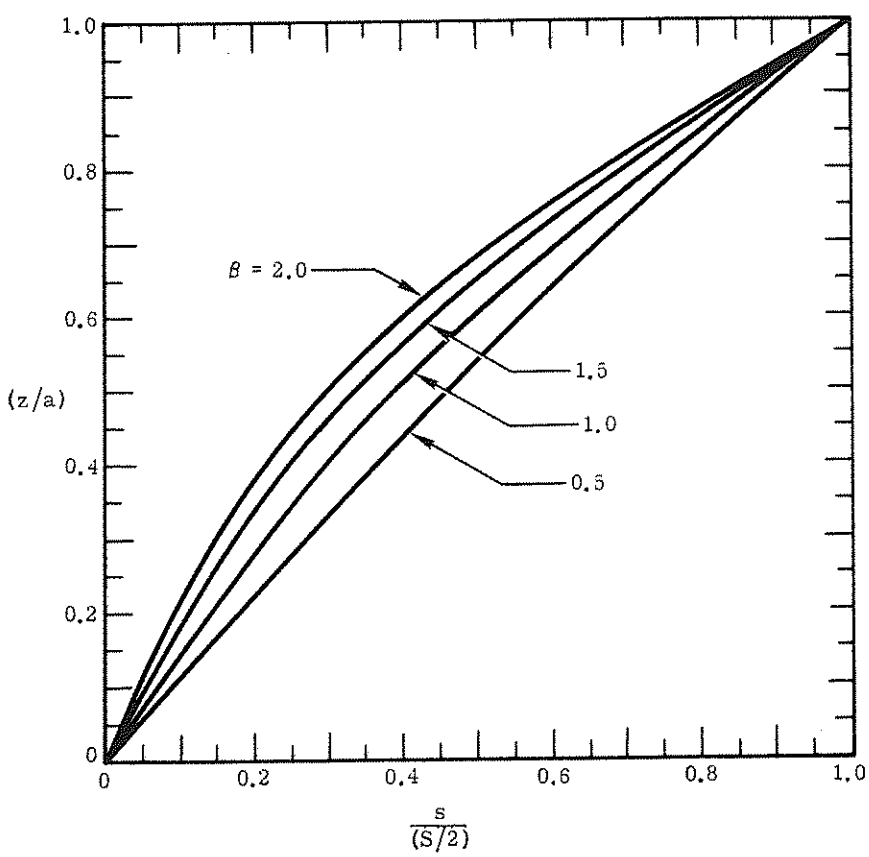

Figure 4. Plots of z/a as a function of s/(S/2) for β's of 0.5, 1.0, 1.5 and 2.0

In dimensionless coordinates the area bounded by the troposkien, the z axis between z = 0 and z = a and the r axis between r = 0 and r = b is given by the integral

$$\frac{A}{ab} = \int_0^1 \left(\frac{z}{a}\right) d\left(\frac{r}{b}\right). \tag{9}$$

and the normalized area swept out by a wind turbine with troposkien blades is

$$\frac{A_s}{ab} = 4 \int_0^1 \left(\frac{z}{a}\right) d\left(\frac{r}{b}\right). \tag{10}$$

Eq. (10) can be integrated analytically, as shown in Appendix C, to yield

$$\frac{A_s}{4ab} = \frac{1}{2kF\left(\frac{\pi}{2};k\right)} \ln\left(\frac{1+k}{1-k}\right). \tag{11}$$

By manipulating Eqs. (3), (7), and (11), it is possible to obtain the following area ratios,

$$\frac{A_s}{4a^2} = \frac{1}{\left(1-k^2\right)F^2\left(\frac{\pi}{2};k\right)} \ln\left(\frac{1+k}{1-k}\right) \tag{12}$$

$$\frac{A_s}{S^2} = \frac{\left(1-k^2\right)\ln\left(\frac{1+k}{1-k}\right)}{\left[2E\left(\frac{\pi}{2};k\right) - \left(1-k^2\right)F\left(\frac{\pi}{2};k\right)\right]^2}. \tag{13}$$

Values of $A_s/(4ab)$, $A_s(4a^2)$ and A_s/S^2 have been calculated as a function of β by means of Eqs. (3), (11), (12), and (13). The results are listed in Appendix A and plotted in Figures 5 and 6.

As can be seen in Figure 6, A_s/S^2 has a maximum in the vicinity of $\beta = 1.0$. It was determined by iterating Eq. (13) that this peak occurs at k = 0.575924 which when inserted into Eq. 3 gives $\beta = 0.994586$. This maximum has practical significance in the choice of β for a wind turbine design and will be discussed later.

In Reference 1 the rotational parameter Ω and the tension parameter k were found to satisfy the equations

$$\Omega = \left[1-k^2\right]^{1/2} F\left(\frac{\pi}{2};k\right) \tag{14}$$

and

$$\left(\frac{T}{T_o}\right)_{max} = \frac{\left(1 + k^2\right)}{\left(1 - k^2\right)} \quad . \tag{15}$$

These two equations have been used in conjunction with Eq. (3) to obtain values of Ω and $(T/T_o)_{max}$ as a function of β. The results are listed in Appendix A and are plotted in Figure 5.

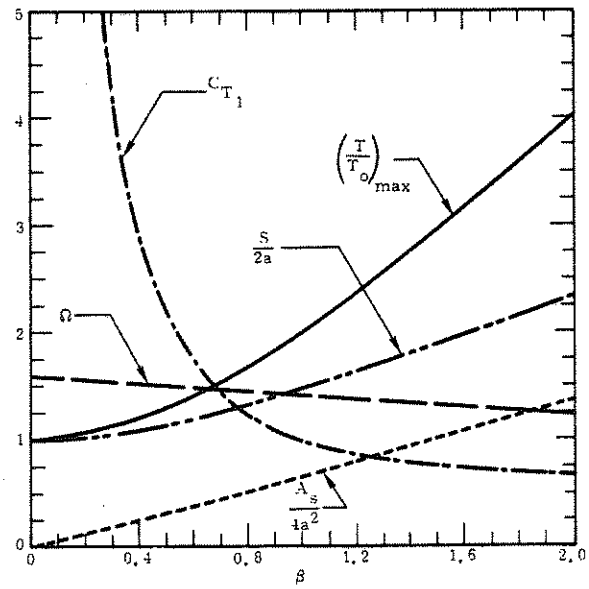

Figure 5. Plots of the ratios Ω, $(T/T_o)_{max}$, C_{T_1}, $S/(2a)$ and $A_s/(4a^2)$ as a function of β between zero and 2.0

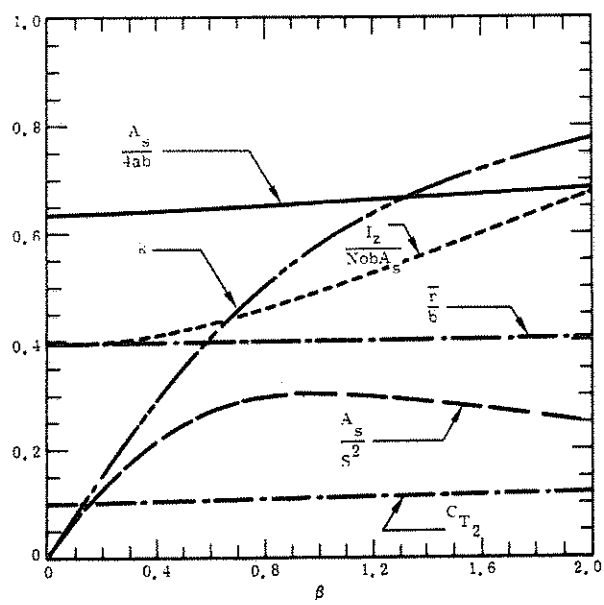

Figure 6. Plots of the ratios k, $\bar{r}/b$, C_{T_2}, $A_s/(4ab)$, $A_s/(S^2)$ and $I_z/(N\delta bA_s)$ as a function of β between zero and 2.0

The quantity T_o, the tension at the point of maximum deflection, can be eliminated from Eq. (15) by substituting from the definition for Ω^2 given in Reference 1,

$$\Omega^2 = \frac{\delta(b\omega)^2}{T_o\beta^2} \tag{16}$$

where it is also shown that

$$\Omega^2\beta^2 = \frac{4k^2}{(1 - k^2)} \quad . \tag{17}$$

Eliminating T_o and $\Omega^2\beta^2$ from these last three equations yields the equation

$$\frac{T_{max}}{\delta(b\omega)^2} = \frac{1 + k^2}{4k^2} \quad . \tag{18}$$

For a troposkien blade of constant cross section A_c and constant mass per unit length δ, it is possible to express δ as

$$\delta = \rho_e A_c \tag{19}$$

where ρ_e is the effective density of the blade. Substituting this last expression for δ into Eq. (18) allows one to define a parameter C_{T_1} as

$$\frac{\left(\frac{T_{max}}{A_c}\right)}{\rho_e(b\omega)^2} = \frac{1 + k^2}{4k^2} = C_{T_1} \quad . \tag{20}$$

which is plotted in Figure 5. This C_{T_1} can be interpreted as a dimensionless tensile stress coefficient since T_{max}/A_c is a tensile stress. This result is important in that it relates how the maximum tensile stress changes as a troposkien type of vertical-axis wind turbine is scaled up or down in size. Since, as shown in Eq. (20), C_{T_1} can be written as a function of k alone, and since from Eq. (3) k can be written as a function of β alone, then C_{T_1} can be expressed as a function of β alone. Therefore, for any given β, C_{T_1} does not vary with the absolute size of the system. Eq. (20) is tabulated in Appendix A and plotted in Figure 5.

If aerodynamic and gravitational loads are neglected, the maximum tensile stress in the blade which is of concern to a troposkien wind turbine designer is that given by

$$\frac{T_{max}}{A_c} = \rho_e(b\omega)^2 C_{T_1} \tag{21}$$

which, for constant β, A_c and ρ_e, varies as $(b\omega)^2$.

At first glance, it may appear that a designer should use some maximum value of the angular velocity ω which is independent of the absolute size of the turbine and that consequently the maximum tensile stress varies as the square of the maximum horizontal displacement b and therefore with the absolute size of the system. However, experience has shown that for any given free-stream wind velocity V, the maximum value of ω is not independent of the absolute size of the system. This maximum ω occurs at the so-called runaway condition which develops when a turbine which is running normally suddenly loses its load and overspeeds. When this happens it has been found that the ratio of the tangential velocity of the blade at its maximum distance from the axis of rotation to the wind velocity is nearly constant at something less than about 8 and almost independent of the detailed design of the system. Therefore, if it is assumed that for maximum tensile stress design purposes

$$\frac{b\omega}{V} = 8, \tag{22}$$

then T_{max}/A_c is <u>independent</u> of the absolute size of the system and varies as the square of the maximum wind velocity. The above statement is strictly true only if the effective density ρ_e is independent of size. In reality, ρ_e will increase with increasing size, but the square dependence on the maximum wind speed will be the controlling factor.

If the length scale b in Eq. (20) is changed to S, the following result, which is developed in Appendix C, can be obtained

$$\frac{\left(\frac{T_{max}}{A_c}\right)}{\rho_e S^2 \omega^2} = \frac{1}{4} \frac{1 + k^2}{\left[2E\left(\frac{\pi}{2};k\right) - \left(1 - k^2\right)F\left(\frac{\pi}{2};k\right)\right]^2} = C_{T_2} \tag{23}$$

where C_{T_2} is a second form of a dimensionless tensile coefficient. Eqs. (20) and (23) are listed in Appendix A and plotted in Figures 5 and 6 as functions of β using, as before, the relationship between k and β given by Eq. (3). As can be seen in Figure 6, there is only a slight variation of C_{T_2} with β.

Two other quantities associated with the troposkien, the normalized moment of inertia about the axis of rotation $I_z/(NSA_s)$ and the corresponding normalized r coordinate of the centroid of the area under a troposkien in the first and fourth quadrants, $\bar{r}/b$, were calculated to be

$$\frac{I_z}{N\delta b A_s} = \frac{\left[(1 - k^2)\,F\left(\frac{\pi}{2};k\right) - (1 + k^2)\,E\left(\frac{\pi}{2};k\right)\right]}{3k(1 - k^2)\,\ell n\left(\frac{1 + k}{1 - k}\right)} \tag{24}$$

and

$$\frac{\bar{r}}{b} = \frac{F\left(\frac{\pi}{2};k\right) - E\left(\frac{\pi}{2};k\right)}{k\ell n\left(\frac{1 + k}{1 - k}\right)} \tag{25}$$

as shown in Appendix C. For a discussion of the I_z normalization factor, see Eq. (C-42). Eqs. (24) and (25) were evaluated numerically, the results of which are listed in Appendix A and plotted in Figure 6.

In order to obtain some comparison between the troposkien and the parabola, those quantities associated with the parabola which correspond to the quantities derived above for the troposkien, are derived in Appendix D.

Observations and Comments

There are several observations which can be made by using the results of the foregoing analyses.

If the swept area is normalized by either a^2 or ab then increasing β increases this normalized swept area. If instead the square of the total blade length is used to normalize the swept area, the maximum value of this normalized area occurs at $\beta = 0.994586$, which can be considered as unity for all practical purposes. This has considerable importance from the standpoint of manufacturing economics. For example, the power output from a given wind turbine varies directly as the swept area if other factors remain the same. Since blade cost goes up with increasing blade length, the maximum power output per total blade length S should occur at $\beta \approx 1$ since A_s/S^2 is a maximum near this point. In actual fact, A_s/S^2 deviates only slightly from the value of 0.3 (with approximately a 2 percent variation) as long as β is in the range 0.8-1.2. As a consequence of this weak dependence of A_s/S^2 on β in this range, some other factor such as aerodynamic performance, tensile stresses at nominal operating conditions or blade sag when the turbine is not rotating could be used to determine an optimum β. As an example consider a blade of fixed total length and compare the tensile stress coefficients C_{T_1} for this blade when $\beta = 0.8$ and 1.2. Using the data tabulated in Appendix A

$$\frac{C_{T_1} \text{ (at } \beta = 0.8)}{C_{T_1} \text{ (at } \rho = 1.2)} \approx 1.5 \, . \tag{26}$$

Since the blade length is considered fixed for this comparison, Eq. (26) when combined with Eq. (20) indicates that at a given rate of rotation ω, the tensile stress is approximately 50 percent greater for the case when $\beta = 0.8$ than it is for the case when $\beta = 1.2$. Using this argument one might choose a design β of 1.2. However, the variation of the aerodynamic performance with β must be determined before a final decision can be made.

The mass moment of inertia of the blades about the axis of rotation is of importance in that it controls the rate of response of the wind turbine to changes in wind speed which in turn influences the overall energy production capability of the wind turbine. As can be seen in Figure 6, as β is increased the dimensionless moment of inertia increases. For practical wind turbine designs it is

felt that the moment of inertia will tend to be large and therefore the small β design should be considered.

Although not all of the quantities derived in this report for the parabola have great physical significance, they are included for the sake of completeness. The numerical results of these calculations are given in Appendix B.

Summary of Equations

Although a number of equations have been derived in this report for the various relationships and characteristics of a troposkien and a troposkien type of wind turbine, they are scattered throughout the report. In order to make them more accessible they are collected here.

Troposkien Coordinates:

$$\frac{z}{a} = 1 - \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)} \tag{1}$$

$$\phi = \sin^{-1}\left(\frac{r}{b}\right) \tag{2}$$

Arc Length:

$$\frac{s}{a} = \frac{2}{(1-k^2)}\left[\frac{E\left(\frac{\pi}{2};k\right) - E(\phi;k)}{F\left(\frac{\pi}{2};k\right)}\right] + \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)} - 1 \tag{6}$$

$$\frac{S}{2a} = \frac{2}{(1-k^2)}\,\frac{E\left(\frac{\pi}{2};k\right)}{F\left(\frac{\pi}{2};k\right)} - 1 \tag{7}$$

$$\frac{s}{\left(\frac{S}{2}\right)} = 1 - \frac{2E(\phi;k) - (1-k^2)F(\phi;k)}{2E\left(\frac{\pi}{2};k\right) - (1-k^2)F\left(\frac{\pi}{2};k\right)} \tag{8}$$

Swept Area:

$$\frac{A_s}{4ab} = \frac{\ln\left(\frac{1+k}{1-k}\right)}{2kF\left(\frac{\pi}{2};k\right)} \tag{11}$$

$$\frac{A_s}{4a^2} = \frac{\ell n\left(\frac{1+k}{1-k}\right)}{(1-k^2)\,F^2\left(\frac{\pi}{2};k\right)} \tag{12}$$

$$\frac{A_s}{S^2} = \frac{(1-k^2)\,\ell n\left(\frac{1+k}{1-k}\right)}{\left[2E\left(\frac{\pi}{2};k\right) - (1-k^2)\,F\left(\frac{\pi}{2};k\right)\right]^2} \tag{13}$$

General Parameters

$$\beta = \frac{2k}{(1-k^2)\,F\left(\frac{\pi}{2};k\right)} \tag{3}$$

$$\Omega = (1-k^2)^{1/2}\,F\left(\frac{\pi}{2};k\right) \tag{14}$$

$$\left(\frac{T}{T_o}\right)_{max} = \frac{1+k^2}{1-k^2} \tag{15}$$

$$\frac{T_{max}}{\delta(b\omega)^2} = \frac{1+k^2}{4k^2} \tag{18}$$

$$\frac{T_{max}}{\delta(\omega S)^2} = \frac{1+k^2}{4\left[2E\left(\frac{\pi}{2};k\right) - (1-k^2)\,F\left(\frac{\pi}{2};k\right)\right]^2} \tag{23}$$

$$\frac{I_z}{N\delta b A_s} = -\frac{(1+k^2)\,E\left(\frac{\pi}{2};k\right) - (1-k^2)\,F\left(\frac{\pi}{2};k\right)}{3k\,(1-k^2)\,\ell n\left(\frac{1+k}{1-k}\right)} \tag{24}$$

$$\frac{\bar{r}}{b} = \frac{F\left(\frac{\pi}{2};k\right) - E\left(\frac{\pi}{2};k\right)}{k\,\ell n\left(\frac{1+k}{1-k}\right)} \tag{25}$$

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3. R. J. Templin, Aerodynamic Performance Theory for the NRC Vertical Axis Wind Turbine, National Research Council of Canada, National Aeronautic Establishment, LTR-LA-160, June 1974.

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APPENDIX A

NORMALIZED COORDINATES OF POINTS LYING ON TROPOSKIENS WITH β's OF 0.25, 0.50, 0.75, 1.0, 1.25, 1.50, 1.75, 2.0, 2.25 AND 2.5

BETA = .250000

K	=	.190951
OMEGA	=	1.556244
(T/TO)MAX	=	1.075684
S/2A	=	1.037666
AS/(4AB)	=	.638588
AS/(4ASQ)	=	.159647
AS/(SSQ)	=	.148267
CT1	=	7.106396
CT2	=	.103123
RBAROB	=	.393307
IZ	=	.400714

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	1.03767	1.00000	1.07568
.96845	.01250	.05000	1.00373	.96729	1.07550
.93682	.02500	.10000	.96972	.93452	1.07493
.90502	.03750	.15000	.93555	.90159	1.07398
.87296	.05000	.20000	.90114	.86843	1.07266
.84056	.06250	.25000	.86641	.83496	1.07095
.80771	.07500	.30000	.83126	.80109	1.06887
.77429	.08750	.35000	.79559	.76671	1.06641
.74018	.10000	.40000	.75925	.73169	1.06357
.70521	.11250	.45000	.72212	.69591	1.06036
.66922	.12500	.50000	.68402	.65919	1.05676
.63196	.13750	.55000	.64472	.62132	1.05279
.59317	.15000	.60000	.60396	.58204	1.04844
.55245	.16250	.65000	.56137	.54100	1.04371
.50932	.17500	.70000	.51646	.49772	1.03860
.46304	.18750	.75000	.46853	.45152	1.03311
.41251	.20000	.80000	.41647	.40135	1.02725
.35586	.21250	.85000	.35845	.34544	1.02100
.28946	.22500	.90000	.29089	.28033	1.01438
.20393	.23750	.95000	.20444	.19702	1.00738
.00000	.25000	1.00000	-.00000	-.00000	1.00000

BETA = .750000

K	=	.482391
OMEGA	=	1.468538
(T/TO)MAX	=	1.606545
S/2A	=	1.293240
AS/(4AB)	=	.650523
AS/(4ASQ)	=	.487892
AS/(SSQ)	=	.291720
CT1	=	1.324341
CT2	=	.111354
RBAROB	=	.397019
IZ	=	.455198

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	1.29324	1.00000	1.60654
.97016	.03750	.05000	1.24532	.96294	1.60503
.94023	.07500	.10000	1.19734	.92584	1.60048
.91011	.11250	.15000	1.14924	.88865	1.59290
.87971	.15000	.20000	1.10096	.85132	1.58228
.84891	.18750	.25000	1.05244	.81380	1.56864
.81761	.22500	.30000	1.00359	.77603	1.55196
.78567	.26250	.35000	.95433	.73794	1.53224
.75295	.30000	.40000	.90456	.69946	1.50950
.71928	.33750	.45000	.85417	.66049	1.48372
.68445	.37500	.50000	.80299	.62091	1.45491
.64822	.41250	.55000	.75084	.58059	1.42307
.61026	.45000	.60000	.69748	.53933	1.38819
.57017	.48750	.65000	.64258	.49688	1.35028
.52738	.52500	.70000	.58568	.45288	1.30934
.48111	.56250	.75000	.52612	.40683	1.26536
.43015	.60000	.80000	.46284	.35789	1.21836
.37247	.63750	.85000	.39404	.30469	1.16832
.30417	.67500	.90000	.31609	.24442	1.11524
.21518	.71250	.95000	.21946	.16970	1.05914
.00000	.75000	1.00000	.00000	.00000	1.00000

BETA = .500000

K	=	.354888
OMEGA	=	1.518384
(T/TO)MAX	=	1.288186
S/2A	=	1.141669
AS/(4AB)	=	.643724
AS/(4ASQ)	=	.321862
AS/(SSQ)	=	.246939
CT1	=	2.234989
CT2	=	.107170
RBAROB	=	.394899
IZ	=	.422867

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	1.14167	1.00000	1.28819
.96920	.02500	.05000	1.10200	.96525	1.28747
.93831	.05000	.10000	1.06226	.93045	1.28530
.90725	.07500	.15000	1.02239	.89552	1.28170
.87591	.10000	.20000	.98230	.86041	1.27666
.84421	.12500	.25000	.94193	.82505	1.27017
.81203	.15000	.30000	.90118	.78935	1.26225
.77925	.17500	.35000	.85996	.75324	1.25288
.74574	.20000	.40000	.81814	.71662	1.24208
.71133	.22500	.45000	.77561	.67936	1.22983
.67582	.25000	.50000	.73219	.64133	1.21614
.63900	.27500	.55000	.68767	.60234	1.20101
.60054	.30000	.60000	.64181	.56216	1.18444
.56007	.32500	.65000	.59424	.52050	1.16643
.51707	.35000	.70000	.54449	.47692	1.14697
.47076	.37500	.75000	.49187	.43083	1.12608
.42002	.40000	.80000	.43529	.38128	1.10375
.36290	.42500	.85000	.37294	.32666	1.07997
.29567	.45000	.90000	.30120	.26382	1.05476
.20865	.47500	.95000	.21063	.18449	1.02810
.00000	.50000	1.00000	-.00000	-.00000	1.00000

BETA = .800000

K	=	.503781
OMEGA	=	1.457985
(T/TO)MAX	=	1.680231
S/2A	=	1.327683
AS/(4AB)	=	.651968
AS/(4ASQ)	=	.521574
AS/(SSQ)	=	.295888
CT1	=	1.235045
CT2	=	.112102
RBAROB	=	.397471
IZ	=	.462550

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	1.32768	1.00000	1.68023
.97036	.04000	.05000	1.27790	.96250	1.67853
.94063	.08000	.10000	1.22806	.92496	1.67343
.91071	.12000	.15000	1.17810	.88734	1.66493
.88049	.16000	.20000	1.12798	.84958	1.65302
.84989	.20000	.25000	1.07761	.81165	1.63772
.81877	.24000	.30000	1.02693	.77348	1.61901
.78701	.28000	.35000	.97586	.73501	1.59690
.75446	.32000	.40000	.92429	.69617	1.57139
.72095	.36000	.45000	.87210	.65686	1.54248
.68627	.40000	.50000	.81916	.61699	1.51017
.65016	.44000	.55000	.76528	.57640	1.47446
.61232	.48000	.60000	.71021	.53492	1.43535
.57231	.52000	.65000	.65363	.49231	1.39283
.52958	.56000	.70000	.59510	.44822	1.34692
.48333	.60000	.75000	.53394	.40216	1.29760
.43233	.64000	.80000	.46912	.35334	1.24488
.37455	.68000	.85000	.39883	.30040	1.18876
.30602	.72000	.90000	.31946	.24062	1.12924
.21661	.76000	.95000	.22144	.16679	1.06632
.00000	.80000	1.00000	-.00000	-.00000	1.00000

BETA = .994586

K = .575924
OMEGA = 1.416654
(T/TO)MAX = 1.992617
S/2A = 1.471360
AS/(4AB) = .657644
AS/(4ASQ) = .654093
AS/(SSQ) = .302131
CT1 = 1.003719
CT2 = .114657
RBAROB = .399254
IZ = .493232

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	1.47136	1.00000	1.99262
.97113	.04973	.05000	1.41386	.96092	1.99014
.94216	.09946	.10000	1.35631	.92181	1.98269
.91300	.14919	.15000	1.29866	.88263	1.97028
.88354	.19892	.20000	1.24086	.84334	1.95291
.85367	.24865	.25000	1.18285	.80391	1.93058
.82327	.29838	.30000	1.12456	.76430	1.90328
.79221	.34810	.35000	1.06593	.72445	1.87102
.76032	.39783	.40000	1.00685	.68430	1.83380
.72744	.44756	.45000	.94724	.64378	1.79161
.69334	.49729	.50000	.88694	.60280	1.74446
.65776	.54702	.55000	.82579	.56124	1.69235
.62037	.59675	.60000	.76357	.51895	1.63527
.58073	.64648	.65000	.69997	.47573	1.57324
.53325	.69621	.70000	.63456	.43128	1.50623
.48209	.74594	.75000	.56671	.38516	1.43427
.44099	.79567	.80000	.49539	.33669	1.35734
.38281	.84540	.85000	.41884	.28466	1.27545
.31344	.89513	.90000	.33345	.22663	1.18860
.22237	.94486	.95000	.22959	.15604	1.09678
.00000	.99459	1.00000	-.00000	-.00000	1.00000

BETA = 1.200000

K = .636113
OMEGA = 1.374020
(T/TO)MAX = 2.359310
S/2A = 1.635751
AS/(4AB) = .663528
AS/(4ASQ) = .796234
AS/(SSQ) = .297582
CT1 = .867834
CT2 = .116763
RBAROB = .401112
IZ = .528350

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	1.63575	1.00000	2.35931
.97191	.06000	.05000	1.56950	.95950	2.35591
.94371	.12000	.10000	1.50321	.91897	2.34572
.91532	.18000	.15000	1.43683	.87839	2.32873
.88662	.24000	.20000	1.37031	.83773	2.30494
.85749	.30000	.25000	1.30362	.79696	2.27435
.82783	.36000	.30000	1.23669	.75604	2.23697
.79748	.42000	.35000	1.16945	.71493	2.19279
.76628	.48000	.40000	1.10182	.67359	2.14182
.73405	.54000	.45000	1.03371	.63195	2.08405
.70057	.60000	.50000	.96500	.58994	2.01948
.66555	.66000	.55000	.89553	.54747	1.94812
.62866	.72000	.60000	.82509	.50441	1.86996
.58944	.78000	.65000	.75341	.46059	1.78500
.54726	.84000	.70000	.68006	.41575	1.69325
.50126	.90000	.75000	.60445	.36953	1.59470
.45011	.96000	.80000	.52560	.32132	1.48935
.39158	1.02000	.85000	.44175	.27006	1.37721
.32139	1.08000	.90000	.34937	.21358	1.25827
.22861	1.14000	.95000	.23875	.14596	1.13253
.00000	1.20000	1.00000	-.00000	-.00000	1.00000

BETA = 1.000000

K = .577703
OMEGA = 1.415510
(T/TO)MAX = 2.001834
S/2A = 1.475545
AS/(4AB) = .657802
AS/(4ASQ) = .657802
AS/(SSQ) = .302127
CT1 = .999085
CT2 = .114719
RBAROB = .399304
IZ = .494126

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	1.47555	1.00000	2.00183
.97115	.05000	.05000	1.41782	.96088	1.99933
.94221	.10000	.10000	1.36005	.92172	1.99182
.91307	.15000	.15000	1.30217	.88250	1.97929
.88363	.20000	.20000	1.24415	.84318	1.96176
.85378	.25000	.25000	1.18592	.80372	1.93922
.82340	.30000	.30000	1.12741	.76406	1.91167
.79235	.35000	.35000	1.06856	.72418	1.87911
.76049	.40000	.40000	1.00927	.68400	1.84154
.72762	.45000	.45000	.94943	.64344	1.79896
.69354	.50000	.50000	.88892	.60243	1.75138
.65797	.55000	.55000	.82756	.56085	1.69878
.62059	.60000	.60000	.76513	.51854	1.64117
.58096	.65000	.65000	.70132	.47530	1.57856
.53849	.70000	.70000	.63572	.43084	1.51094
.49234	.75000	.75000	.56767	.38472	1.43830
.44123	.80000	.80000	.49616	.33626	1.36066
.38304	.85000	.85000	.41942	.28425	1.27801
.31365	.90000	.90000	.33386	.22626	1.19035
.22253	.95000	.95000	.22983	.15576	1.09768
.00000	1.00000	1.00000	-.00000	-.00000	1.00000

BETA = 1.250000

K = .648736
OMEGA = 1.363941
(T/TO)MAX = 2.453386
S/2A = 1.677269
AS/(4AB) = .664924
AS/(4ASQ) = .831155
AS/(SSQ) = .295445
CT1 = .844024
CT2 = .117195
RBAROB = .401554
IZ = .537220

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	1.67727	1.00000	2.45339
.97209	.06250	.05000	1.60882	.95919	2.44975
.94407	.12500	.10000	1.54033	.91835	2.43885
.91586	.18750	.15000	1.47175	.87747	2.42068
.88733	.25000	.20000	1.40305	.83651	2.39525
.85839	.31250	.25000	1.33417	.79544	2.36255
.82889	.37500	.30000	1.26506	.75424	2.32258
.79871	.43750	.35000	1.19566	.71286	2.27535
.76767	.50000	.40000	1.12588	.67126	2.22084
.73560	.56250	.45000	1.05563	.62937	2.15908
.70227	.62500	.50000	.98479	.58714	2.09004
.66739	.68750	.55000	.91322	.54447	2.01374
.63062	.75000	.60000	.84070	.50123	1.93017
.59150	.81250	.65000	.76696	.45727	1.83933
.54940	.87500	.70000	.69161	.41234	1.74123
.50345	.93750	.75000	.61403	.36609	1.63586
.45229	1.00000	.80000	.53325	.31793	1.52322
.39369	1.06250	.85000	.44755	.26683	1.40331
.32331	1.12500	.90000	.35338	.21069	1.27614
.23012	1.18750	.95000	.24104	.14371	1.14171
.00000	1.25000	1.00000	-.00000	-.00000	1.00000

BETA = 1.500000

K = .702393
OMEGA = 1.315733
(T/TO)MAX = 2.947546
S/2A = 1.891502
AS/(4AB) = .671621
AS/(4ASQ) = 1.007431
AS/(SSQ) = .281580
CT1 = .756733
CT2 = .118974
RBAROB = .403683
IZ = .582982

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	1.89150	1.00000	2.94755
.97293	.07500	.05000	1.81177	.95785	2.94268
.94577	.15000	.10000	1.73200	.91567	2.92807
.91839	.22500	.15000	1.65216	.87346	2.90373
.89071	.30000	.20000	1.57221	.83120	2.86964
.86259	.37500	.25000	1.49211	.78885	2.82582
.83391	.45000	.30000	1.41182	.74640	2.77227
.80452	.52500	.35000	1.33127	.70381	2.70897
.77427	.60000	.40000	1.25039	.66106	2.63594
.74295	.67500	.45000	1.16912	.61809	2.55317
.71033	.75000	.50000	1.08733	.57485	2.46066
.67612	.82500	.55000	1.00490	.53127	2.35841
.63996	.90000	.60000	.92163	.48725	2.24643
.60137	.97500	.65000	.83728	.44265	2.12471
.55970	1.05000	.70000	.75148	.39729	1.99325
.51401	1.12500	.75000	.66365	.35086	1.85205
.46289	1.20000	.80000	.57287	.30287	1.70112
.40398	1.27500	.85000	.47748	.25244	1.54044
.33275	1.35000	.90000	.37399	.19772	1.37003
.23764	1.42500	.95000	.25271	.13360	1.18989
.00000	1.50000	1.00000	-.00000	-.00000	1.00000

BETA = 2.000000

K = .776156
OMEGA = 1.230935
(T/TO)MAX = 4.030404
S/2A = 2.342409
AS/(4AB) = .683492
AS/(4ASQ) = 1.366983
AS/(SSQ) = .249137
CT1 = .664994
CT2 = .121197
RBAROB = .407481
IZ = .679206

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	2.34241	1.00000	4.03040
.97437	.10000	.05000	2.23918	.95593	4.02283
.94864	.20000	.10000	2.13592	.91185	4.00010
.92270	.30000	.15000	2.03261	.86774	3.96222
.89643	.40000	.20000	1.92922	.82360	3.90919
.86973	.50000	.25000	1.82571	.77942	3.84100
.84246	.60000	.30000	1.72206	.73517	3.75767
.81446	.70000	.35000	1.61822	.69083	3.65918
.78558	.80000	.40000	1.51413	.64640	3.54554
.75559	.90000	.45000	1.40973	.60183	3.41675
.72427	1.00000	.50000	1.30494	.55709	3.27280
.69130	1.10000	.55000	1.19964	.51214	3.11371
.65630	1.20000	.60000	1.09369	.46691	2.93946
.61875	1.30000	.65000	.98687	.42131	2.75006
.57795	1.40000	.70000	.87886	.37520	2.54551
.53291	1.50000	.75000	.76918	.32837	2.32580
.48207	1.60000	.80000	.65699	.28047	2.09095
.42288	1.70000	.85000	.54075	.23085	1.84094
.35037	1.80000	.90000	.41716	.17809	1.57578
.25194	1.90000	.95000	.27663	.11810	1.29546
.00000	2.00000	1.00000	-.00000	-.00000	1.00000

BETA = 1.750000

K = .743707
OMEGA = 1.271419
(T/TO)MAX = 3.475277
S/2A = 2.114074
AS/(4AB) = .677810
AS/(4ASQ) = 1.186167
AS/(SSQ) = .265403
CT1 = .701998
CT2 = .120257
RBAROB = .405659
IZ = .630500

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	2.11407	1.00000	3.47528
.97369	.08750	.05000	2.02271	.95678	3.46909
.94728	.17500	.10000	1.93131	.91355	3.45052
.92067	.26250	.15000	1.83985	.87029	3.41958
.89373	.35000	.20000	1.74830	.82698	3.37627
.86636	.43750	.25000	1.65661	.78361	3.32057
.83842	.52500	.30000	1.56476	.74016	3.25250
.80976	.61250	.35000	1.47269	.69661	3.17206
.78022	.70000	.40000	1.38034	.65293	3.07923
.74960	.78750	.45000	1.28763	.60908	2.97403
.71765	.87500	.50000	1.19448	.56501	2.85646
.68409	.96250	.55000	1.10076	.52068	2.72651
.64852	1.05000	.60000	1.00631	.47600	2.58418
.61045	1.13750	.65000	.91088	.43087	2.42947
.56922	1.22500	.70000	.81415	.38511	2.26239
.52383	1.31250	.75000	.71557	.33848	2.08293
.47283	1.40000	.80000	.61428	.29057	1.89110
.41373	1.48750	.85000	.50867	.24061	1.68689
.34180	1.57500	.90000	.39533	.18700	1.47030
.24494	1.66250	.95000	.26461	.12516	1.24134
.00000	1.75000	1.00000	-.00000	-.00000	1.00000

BETA = 2.250000

K = .802123
OMEGA = 1.193945
(T/TO)MAX = 4.608550
S/2A = 2.574845
AS/(4AB) = .688701
AS/(4ASQ) = 1.549576
AS/(SSQ) = .233724
CT1 = .638560
CT2 = .121898
RBAROB = .409157
IZ = .728737

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	2.57486	1.00000	4.60855
.97498	.11250	.05000	2.45962	.95524	4.59953
.94985	.22500	.10000	2.34434	.91047	4.57246
.92451	.33750	.15000	2.22903	.86569	4.52736
.89885	.45000	.20000	2.11364	.82087	4.46421
.87275	.56250	.25000	1.99815	.77602	4.38302
.84608	.67500	.30000	1.88253	.73112	4.28378
.81869	.78750	.35000	1.76674	.68615	4.16650
.79039	.90000	.40000	1.65074	.64110	4.03118
.76099	1.01250	.45000	1.53446	.59594	3.87782
.73025	1.12500	.50000	1.41783	.55064	3.70641
.69784	1.23750	.55000	1.30076	.50517	3.51696
.66337	1.35000	.60000	1.18309	.45948	3.30947
.62631	1.46250	.65000	1.06465	.41348	3.08394
.58596	1.57500	.70000	.94512	.36706	2.84036
.54126	1.68750	.75000	.82406	.32004	2.57874
.49064	1.80000	.80000	.70068	.27212	2.29908
.43142	1.91250	.85000	.57352	.22274	2.00137
.35845	2.02500	.90000	.43936	.17063	1.68562
.25863	2.13750	.95000	.28871	.11213	1.35183
.00000	2.25000	1.00000	-.00000	-.00000	1.00000

BETA = 2.500000

K	=	.823260
OMEGA	=	1.160205
(T/TO)MAX	=	5.206488
S/2A	=	2.810357
AS/(4AB)	=	.693482
AS/(4ASQ)	=	1.733706
AS/(SSQ)	=	.219509
CT1	=	.618864
CT2	=	.122431
RBAROB	=	.410701
IZ	=	.778854

Z/A	R/A	R/B	s/A	s/(S/2)	T/TO
1.00000	0.00000	0.00000	2.81036	1.00000	5.20649
.97552	.12500	.05000	2.68298	.95468	5.19597
.95093	.25000	.10000	2.55559	.90935	5.16442
.92614	.37500	.15000	2.42815	.86400	5.11184
.90103	.50000	.20000	2.30065	.81863	5.03823
.87547	.62500	.25000	2.17307	.77324	4.94358
.84934	.75000	.30000	2.04537	.72780	4.82790
.82249	.87500	.35000	1.91751	.68230	4.69119
.79473	1.00000	.40000	1.78947	.63674	4.53345
.76587	1.12500	.45000	1.66118	.59109	4.35467
.73565	1.25000	.50000	1.53258	.54533	4.15487
.70376	1.37500	.55000	1.40357	.49943	3.93403
.66980	1.50000	.60000	1.27404	.45334	3.69215
.63322	1.62500	.65000	1.14380	.40699	3.42925
.59329	1.75000	.70000	1.01257	.36030	3.14531
.54896	1.87500	.75000	.87993	.31310	2.84034
.49857	2.00000	.80000	.74515	.26514	2.51434
.43939	2.12500	.85000	.60682	.21592	2.16730
.36606	2.25000	.90000	.46182	.16433	1.79923
.26501	2.37500	.95000	.30082	.10704	1.41013
.00000	2.50000	1.00000	.00000	.00000	1.00000

Note: These are computer generated tables. The labels used in these tables are made up from symbols available on the printer.

APPENDIX B

LIST OF PARAMETERS AND COORDINATES ASSOCIATED WITH PARABOLAS DEFINED BY $(z/a)^2 = 1 - (1/8)(r/a)$ FOR β's OF 0.25, 0.50, 0.75, 1.0, 1.25, 1.50, 1.75, 2.0, 2.25 AND 2.5

BETA = .250000

S/2A = 1.040229
AS/(4AB) = .666667
AS/(4ASQ) = .166667
AS/(SSQ) = .154025
RBAROB = .400000
IZ = .407002

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	.25000	1.00000	0.00000	0.00000
.05000	.24937	.99750	.05001	.04807
.10000	.24750	.99000	.10004	.09617
.15000	.24437	.97750	.15014	.14433
.20000	.24000	.96000	.20033	.19259
.25000	.23438	.93750	.25065	.24096
.30000	.22750	.91000	.30112	.28948
.35000	.21937	.87750	.35178	.33817
.40000	.21000	.84000	.40265	.38708
.45000	.19937	.79750	.45377	.43622
.50000	.18750	.75000	.50516	.48562
.55000	.17437	.69750	.55686	.53532
.60000	.16000	.64000	.60888	.58533
.65000	.14438	.57750	.66127	.63569
.70000	.12750	.51000	.71404	.68643
.75000	.10938	.43750	.76722	.73755
.80000	.09000	.36000	.82085	.78910
.85000	.06938	.27750	.87494	.84110
.90000	.04750	.19000	.92951	.89357
.95000	.02438	.09750	.98460	.94653
1.00000	0.00000	0.00000	1.04023	1.00000

BETA = .500000

S/2A = 1.147794
AS/(4AB) = .666667
AS/(4ASQ) = .333333
AS/(SSQ) = .253018
RBAROB = .400000
IZ = .426602

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	.50000	1.00000	0.00000	0.00000
.05000	.49875	.99750	.05002	.04358
.10000	.49500	.99000	.10017	.08727
.15000	.48875	.97750	.15056	.13117
.20000	.48000	.96000	.20133	.17540
.25000	.46875	.93750	.25258	.22006
.30000	.45500	.91000	.30444	.26524
.35000	.43875	.87750	.35702	.31105
.40000	.42000	.84000	.41042	.35758
.45000	.39875	.79750	.46476	.40491
.50000	.37500	.75000	.52011	.45314
.55000	.34875	.69750	.57659	.50235
.60000	.32000	.64000	.63427	.55260
.65000	.28875	.57750	.69324	.60397
.70000	.25500	.51000	.75356	.65653
.75000	.21875	.43750	.81532	.71034
.80000	.18000	.36000	.87858	.76545
.85000	.13875	.27750	.94341	.82193
.90000	.09500	.19000	1.00985	.87982
.95000	.04875	.09750	1.07796	.93916
1.00000	0.00000	0.00000	1.14779	1.00000

BETA = .750000

S/2A = 1.299642
AS/(4AB) = .666667
AS/(4ASQ) = .500000
AS/(SSQ) = .296021
RBAROB = .400000
IZ = .455903

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	.75000	1.00000	0.00000	0.00000
.05000	.74812	.99750	.05005	.03851
.10000	.74250	.99000	.10037	.07723
.15000	.73312	.97750	.15126	.11638
.20000	.72000	.96000	.20296	.15617
.25000	.70313	.93750	.25574	.19678
.30000	.68250	.91000	.30984	.23840
.35000	.65812	.87750	.36547	.28121
.40000	.63000	.84000	.42285	.32536
.45000	.59812	.79750	.48215	.37099
.50000	.56250	.75000	.54355	.41823
.55000	.52312	.69750	.60720	.46720
.60000	.48000	.64000	.67323	.51801
.65000	.43313	.57750	.74177	.57075
.70000	.38250	.51000	.81293	.62550
.75000	.32813	.43750	.88680	.68234
.80000	.27000	.36000	.96348	.74134
.85000	.20813	.27750	1.04303	.80255
.90000	.14250	.19000	1.12554	.86604
.95000	.07313	.09750	1.21106	.93184
1.00000	0.00000	0.00000	1.29964	1.00000

BETA = .800000

S/2A = 1.333705
AS/(4AB) = .666667
AS/(4ASQ) = .533333
AS/(SSQ) = .299833
RBAROB = .400000
IZ = .462681

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	.80000	1.00000	0.00000	0.00000
.05000	.79800	.99750	.05005	.03753
.10000	.79200	.99000	.10043	.07530
.15000	.78200	.97750	.15143	.11354
.20000	.76800	.96000	.20336	.15248
.25000	.75000	.93750	.25652	.19233
.30000	.72800	.91000	.31115	.23330
.35000	.70200	.87750	.36752	.27556
.40000	.67200	.84000	.42583	.31929
.45000	.63800	.79750	.48631	.36463
.50000	.60000	.75000	.54912	.41172
.55000	.55800	.69750	.61442	.46069
.60000	.51200	.64000	.68237	.51163
.65000	.46200	.57750	.75308	.56465
.70000	.40800	.51000	.82668	.61984
.75000	.35000	.43750	.90326	.67726
.80000	.28800	.36000	.98291	.73698
.85000	.22200	.27750	1.06572	.79906
.90000	.15200	.19000	1.15174	.86357
.95000	.07800	.09750	1.24105	.93053
1.00000	0.00000	0.00000	1.33371	1.00000

BETA = .994586

S/2A = 1.474848
AS/(4AB) = .666667
AS/(4ASQ) = .663057
AS/(SSQ) = .304829
RBAROB = .400000
IZ = .491396

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	.99459	1.00000	0.00000	0.00000
.05000	.99210	.99750	.05008	.03396
.10000	.98464	.99000	.10066	.06825
.15000	.97221	.97750	.15220	.10319
.20000	.95480	.96000	.20516	.13910
.25000	.93242	.93750	.25995	.17626
.30000	.90507	.91000	.31696	.21491
.35000	.87275	.87750	.37651	.25529
.40000	.83545	.84000	.43890	.29759
.45000	.79318	.79750	.50438	.34199
.50000	.74594	.75000	.57318	.38863
.55000	.69372	.69750	.64548	.43766
.60000	.63653	.64000	.72145	.48917
.65000	.57437	.57750	.80123	.54326
.70000	.50724	.51000	.88494	.60002
.75000	.43513	.43750	.97269	.65952
.80000	.35805	.36000	1.06457	.72182
.85000	.27600	.27750	1.16066	.78697
.90000	.18897	.19000	1.26103	.85502
.95000	.09697	.09750	1.36574	.92602
1.00000	0.00000	0.00000	1.47485	1.00000

BETA = 1.200000

S/2A = 1.635300
AS/(4AB) = .666667
AS/(4ASQ) = .800000
AS/(SSQ) = .299154
RBAROB = .400000
IZ = .525049

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	1.20000	1.00000	0.00000	0.00000
.05000	1.19700	.99750	.05012	.03065
.10000	1.18800	.99000	.10095	.06173
.15000	1.17300	.97750	.15318	.09367
.20000	1.15200	.96000	.20743	.12685
.25000	1.12500	.93750	.26428	.16161
.30000	1.09200	.91000	.32420	.19825
.35000	1.05300	.87750	.38763	.23704
.40000	1.00800	.84000	.45491	.27818
.45000	.95700	.79750	.52634	.32186
.50000	.90000	.75000	.60217	.36823
.55000	.83700	.69750	.68261	.41742
.60000	.76800	.64000	.76783	.46953
.65000	.69300	.57750	.85797	.52466
.70000	.61200	.51000	.95317	.58287
.75000	.52500	.43750	1.05351	.64423
.80000	.43200	.36000	1.15911	.70880
.85000	.33300	.27750	1.27002	.77663
.90000	.22800	.19000	1.38632	.84775
.95000	.11700	.09750	1.50806	.92219
1.00000	0.00000	0.00000	1.63530	1.00000

BETA = 1.000000

S/2A = 1.478943
AS/(4AB) = .666667
AS/(4ASQ) = .666667
AS/(SSQ) = .304794
RBAROB = .400000
IZ = .492243

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	1.00000	1.00000	0.00000	0.00000
.05000	.99750	.99750	.05008	.03386
.10000	.99000	.99000	.10066	.06806
.15000	.97750	.97750	.15222	.10293
.20000	.96000	.96000	.20521	.13876
.25000	.93750	.93750	.26006	.17584
.30000	.91000	.91000	.31713	.21443
.35000	.87750	.87750	.37678	.25476
.40000	.84000	.84000	.43929	.29703
.45000	.79750	.79750	.50492	.34141
.50000	.75000	.75000	.57390	.38805
.55000	.69750	.69750	.64640	.43707
.60000	.64000	.64000	.72261	.48860
.65000	.57750	.57750	.80265	.54272
.70000	.51000	.51000	.88666	.59952
.75000	.43750	.43750	.97473	.65907
.80000	.36000	.36000	1.06696	.72144
.85000	.27750	.27750	1.16344	.78667
.90000	.19000	.19000	1.26422	.85481
.95000	.09750	.09750	1.36937	.92591
1.00000	0.00000	0.00000	1.47894	1.00000

BETA = 1.250000

S/2A = 1.675737
AS/(4AB) = .666667
AS/(4ASQ) = .833333
AS/(SSQ) = .296761
RBAROB = .400000
IZ = .533669

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	1.25000	1.00000	0.00000	0.00000
.05000	1.24687	.99750	.05013	.02992
.10000	1.23750	.99000	.10103	.06029
.15000	1.22187	.97750	.15344	.09157
.20000	1.20000	.96000	.20805	.12415
.25000	1.17188	.93750	.26543	.15840
.30000	1.13750	.91000	.32613	.19462
.35000	1.09687	.87750	.39057	.23307
.40000	1.05000	.84000	.45912	.27398
.45000	.99687	.79750	.53208	.31752
.50000	.93750	.75000	.60971	.36385
.55000	.87187	.69750	.69222	.41309
.60000	.80000	.64000	.77979	.46534
.65000	.72188	.57750	.87255	.52069
.70000	.63750	.51000	.97063	.57922
.75000	.54688	.43750	1.07413	.64099
.80000	.45000	.36000	1.18315	.70605
.85000	.34688	.27750	1.29776	.77444
.90000	.23750	.19000	1.41803	.84621
.95000	.12188	.09750	1.54400	.92139
1.00000	0.00000	0.00000	1.67574	1.00000

BETA = 1.500000

S/2A = 1.884213
AS/(4AB) = .666667
AS/(4ASQ) = 1.000000
AS/(SSQ) = .281670
RBAROB = .400000
IZ = .578839

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	1.50000	1.00000	0.00000	0.00000
.05000	1.49625	.99750	.05019	.02664
.10000	1.48500	.99000	.10148	.05386
.15000	1.46625	.97750	.15492	.08222
.20000	1.44000	.96000	.21142	.11221
.25000	1.40625	.93750	.27177	.14424
.30000	1.36500	.91000	.33662	.17865
.35000	1.31625	.87750	.40647	.21572
.40000	1.26000	.84000	.48174	.25567
.45000	1.19625	.79750	.56277	.29868
.50000	1.12500	.75000	.64982	.34488
.55000	1.04625	.69750	.74311	.39439
.60000	.96000	.64000	.84281	.44730
.65000	.86625	.57750	.94907	.50369
.70000	.76500	.51000	1.06199	.56363
.75000	.65625	.43750	1.18169	.62715
.80000	.54000	.36000	1.30824	.69432
.85000	.41625	.27750	1.44171	.76515
.90000	.28500	.19000	1.58216	.83970
.95000	.14625	.09750	1.72965	.91797
1.00000	0.00000	0.00000	1.88421	1.00000

BETA = 2.000000

S/2A = 2.323392
AS/(4AB) = .666667
AS/(4ASQ) = 1.333333
AS/(SSQ) = .246998
RBAROB = .400000
IZ = .676981

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	2.00000	1.00000	0.00000	0.00000
.05000	1.99500	.99750	.05033	.02166
.10000	1.98000	.99000	.10261	.04416
.15000	1.95500	.97750	.15857	.06825
.20000	1.92000	.96000	.21965	.09454
.25000	1.87500	.93750	.28695	.12350
.30000	1.82000	.91000	.36130	.15551
.35000	1.75500	.87750	.44333	.19081
.40000	1.68000	.84000	.53348	.22961
.45000	1.59500	.79750	.63211	.27206
.50000	1.50000	.75000	.73947	.31827
.55000	1.39500	.69750	.85578	.36833
.60000	1.28000	.64000	.98118	.42230
.65000	1.15500	.57750	1.11581	.48025
.70000	1.02000	.51000	1.25978	.54222
.75000	.87500	.43750	1.41316	.60823
.80000	.72000	.36000	1.57603	.67833
.85000	.55500	.27750	1.74844	.75254
.90000	.38000	.19000	1.93044	.83087
.95000	.19500	.09750	2.12208	.91336
1.00000	0.00000	0.00000	2.32339	1.00000

BETA = 1.750000

S/2A = 2.100845
AS/(4AB) = .666667
AS/(4ASQ) = 1.166667
AS/(SSQ) = .264338
RBAROB = .400000
IZ = .626825

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	1.75000	1.00000	0.00000	0.00000
.05000	1.74562	.99750	.05025	.02392
.10000	1.73250	.99000	.10201	.04855
.15000	1.71062	.97750	.15663	.07456
.20000	1.68000	.96000	.21530	.10248
.25000	1.64063	.93750	.27898	.13279
.30000	1.59250	.91000	.34840	.16584
.35000	1.53562	.87750	.42415	.20189
.40000	1.47000	.84000	.50666	.24117
.45000	1.39562	.79750	.59629	.28383
.50000	1.31250	.75000	.69331	.33001
.55000	1.22062	.69750	.79791	.37981
.60000	1.12000	.64000	.91028	.43329
.65000	1.01062	.57750	1.03055	.49054
.70000	.89250	.51000	1.15882	.55160
.75000	.76563	.43750	1.29520	.61651
.80000	.63000	.36000	1.43975	.68532
.85000	.48563	.27750	1.59254	.75805
.90000	.33250	.19000	1.75362	.83472
.95000	.17063	.09750	1.92304	.91537
1.00000	0.00000	0.00000	2.10084	1.00000

BETA = 2.250000

S/2A = 2.550369
AS/(4AB) = .666667
AS/(4ASQ) = 1.500000
AS/(SSQ) = .230614
RBAROB = .400000
IZ = .728840

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	2.25000	1.00000	0.00000	0.00000
.05000	2.24437	.99750	.05042	.01977
.10000	2.22750	.99000	.10328	.04050
.15000	2.19937	.97750	.16072	.06302
.20000	2.16000	.96000	.22441	.08799
.25000	2.10938	.93750	.29560	.11591
.30000	2.04750	.91000	.37518	.14711
.35000	1.97437	.87750	.46378	.18185
.40000	1.89000	.84000	.56187	.22031
.45000	1.79437	.79750	.66979	.26263
.50000	1.68750	.75000	.78779	.30889
.55000	1.56937	.69750	.91607	.35919
.60000	1.44000	.64000	1.05478	.41358
.65000	1.29937	.57750	1.20403	.47210
.70000	1.14750	.51000	1.36393	.53480
.75000	.98438	.43750	1.53455	.60170
.80000	.81000	.36000	1.71595	.67282
.85000	.62438	.27750	1.90819	.74820
.90000	.42750	.19000	2.11132	.82785
.95000	.21938	.09750	2.32537	.91178
1.00000	0.00000	0.00000	2.55037	1.00000

BETA = 2.500000

S/2A = 2.780754
AS/(4AB) = .666667
AS/(4ASQ) = 1.666667
AS/(SSQ) = .215538
RBAROB = .400000
IZ = .782062

Z/A	R/A	R/B	s/A	s/(S/2)
0.00000	2.50000	1.00000	0.00000	0.00000
.05000	2.49375	.99750	.05052	.01817
.10000	2.47500	.99000	.10402	.03741
.15000	2.44375	.97750	.16306	.05864
.20000	2.40000	.96000	.22956	.08255
.25000	2.34375	.93750	.30486	.10963
.30000	2.27500	.91000	.38989	.14021
.35000	2.19375	.87750	.48531	.17453
.40000	2.10000	.84000	.59158	.21274
.45000	1.99375	.79750	.70901	.25497
.50000	1.87500	.75000	.83787	.30131
.55000	1.74375	.69750	.97833	.35182
.60000	1.60000	.64000	1.13053	.40655
.65000	1.44375	.57750	1.29459	.46555
.70000	1.27500	.51000	1.47059	.52885
.75000	1.09375	.43750	1.65861	.59646
.80000	.90000	.36000	1.85871	.66842
.85000	.69375	.27750	2.07094	.74474
.90000	.47500	.19000	2.29533	.82544
.95000	.24375	.09750	2.53193	.91052
1.00000	0.00000	0.00000	2.78075	1.00000

Note: These are computer generated tables. The labels used in these tables are made up from symbols available on the printer.

APPENDIX C

DETAILS OF THE ANALYSIS

The procedure for calculating k for a given β is to start with Eq. (3) in the body of this report and rewrite it in the form

$$G = 2k - \beta(1 - k^2)F\left(\frac{\pi}{2};k\right). \tag{C1}$$

For a given β, k is varied until G is less than some prescribed value. For the work reported here the final values of G were required to be $\leq 10^{-13}$. The actual calculations were done in a series of subroutines in the computer code TROPA using the standard Newton-Raphson technique.

In order to calculate the variables in Eq.'s (6), (8), (11), (12) and (13) in the body of the report, it is necessary to derive expressions for the troposkien arc lengths and the swept area A_s.

In Ref. 1, the equation relating the incremental troposkien arc length and the z coordinate is given as

$$\frac{d(s/a)}{d(z/a)} = 1 - \frac{\Omega^2}{2}\left[\left(\frac{r}{a}\right)^2 - \beta^2\right] \tag{C2}$$

Integrating this gives

$$\frac{s}{a} = \int_0^{z/a} \left(1 - \frac{\Omega^2\beta^2}{2}\left[\left(\frac{r}{b}\right)^2 - 1\right]\right) d\left(\frac{z}{a}\right). \tag{C3}$$

The constant terms can be evaluated directly; however, the term containing $\left(\frac{r}{b}\right)^2$ is a little involved. Taking the expression for (z/a) from Ref. 1,

$$\frac{z}{a} = \frac{1}{\Omega}\sqrt{1 - k^2}\left[F\left(\frac{\pi}{2};k\right) - F(\phi;k)\right] \tag{C4}$$

from which

$$\frac{d\left(\frac{z}{a}\right)}{d\phi} = -\frac{\sqrt{1 - k^2}}{\Omega}\frac{dF(\phi;k)}{d\phi} = -\frac{\sqrt{1 - k^2}}{\Omega\sqrt{1 - k^2 \sin^2 \phi}}. \tag{C5}$$

In these expressions, the quantity ϕ is defined in Eq. (2),

$$\sin \phi = \left(\frac{r}{b}\right). \tag{C6}$$

Substituting for $d\left(\frac{z}{a}\right)$ from Eq. (C5) and $\left(\frac{r}{b}\right)$ from Eq. (C6) into Eq. (A3)

$$\frac{s}{a} = \left(1 + \frac{\Omega^2\beta^2}{2}\right)\frac{z}{a} + \frac{\Omega\beta^2\sqrt{1 - k^2}}{2k^2}\int_{\pi/2}^{\sin^{-1}\left(\frac{r}{b}\right)} \frac{k^2 \sin^2 \phi}{\sqrt{1 - k^2 \sin^2 \phi}}\, d\phi. \tag{C7}$$

Substituting for $\left(\frac{z}{a}\right)$ from Eq. (C4) and evaluating the integral[4],

$$\frac{s}{a} = \left(1 + \frac{\Omega^2\beta^2}{2}\right)\frac{1}{\Omega}\sqrt{1 - k^2}\left[F\left(\frac{\pi}{2};k\right) - F(\phi;k)\right]$$

$$- \frac{\Omega^2\beta^2}{2}\frac{\sqrt{1 - k^2}}{\Omega k^2}\left[F\left(\frac{\pi}{2};k\right) - F(\phi;k) - E\left(\frac{\pi}{2};k\right) + E(\phi;k)\right]. \tag{C8}$$

From Eq. (A19) in reference 1,

$$\frac{\sqrt{1 - k^2}}{\Omega} = \frac{1}{F\left(\frac{\pi}{2};k\right)}. \tag{C9}$$

Substituting this into Eq. (C8),

$$\frac{s}{a} = \left(1 + \frac{\Omega^2\beta^2}{2}\right)\left[1 - \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)}\right] - \frac{\Omega^2\beta^2}{2k^2}\left[\left(1 - \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)}\right) - \left(\frac{E\left(\frac{\pi}{2};k\right) - E(\phi;k)}{F\left(\frac{\pi}{2};k\right)}\right)\right]$$

$$= \left[1 - \frac{\Omega^2\beta^2(1 - k^2)}{2k^2}\right]\left[1 - \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)}\right] + \frac{\Omega^2\beta^2}{2k^2F\left(\frac{\pi}{2};k\right)}\left[E\left(\frac{\pi}{2};k\right) - E(\phi;k)\right]. \tag{C10}$$

From Eq. (A13) in Ref. 1,

$$\frac{\Omega^2\beta^2(1 - k^2)}{2k^2} = 2. \tag{C11}$$

Substituting this into Eq. (C10),

$$\frac{s}{a} = \frac{2}{(1 - k^2)F\left(\frac{\pi}{2};k\right)}\left[E\left(\frac{\pi}{2};k\right) - E(\phi;k)\right] + \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)} - 1 \tag{C12}$$

This is Eq. (6) in the body of the paper. The normalized troposkien length S/a is obtained by evaluating Eq. (C12), at the end point $\phi = 0$.

$$\frac{S}{a} = \frac{2s}{a}\bigg]_{\phi=0} = \frac{4E\left(\frac{\pi}{2};k\right)}{(1 - k^2)F\left(\frac{\pi}{2};k\right)} - 2. \tag{C13}$$

The swept area A_s is given in integral form in Ref. 1 as

$$A_s = 4ab\int_0^{\pi/2} \cos\phi\left[1 - \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)}\right]d\phi. \tag{C14}$$

This can be separated into

$$A_s = 4ab - \frac{4ab}{F\left(\frac{\pi}{2};k\right)} \int_0^{\pi/2} \cos\phi\, F(\phi;k)\, d\phi. \tag{C15}$$

The integral in this last equation can be integrated by parts.

$$\text{Let} \quad \left.\begin{aligned} u &= \sin\phi & du &= \cos\phi\, d\phi \\ v &= F(\phi;k) & dv &= \frac{d\phi}{\sqrt{1 - k^2 \sin^2\phi}} \end{aligned}\right\} \tag{C16}$$

Then, since

$$\int v\,du = uv - \int u\,dv \tag{C17}$$

$$\int_0^{\pi/2} \cos\phi\, F(\phi;k)\, d\phi = \sin\phi\, F(\phi;k)\Big|_0^{\pi/2} - \int_0^{\pi/2} \frac{\sin\phi}{\sqrt{1 - k^2 \sin^2\phi}}\, d\phi \tag{C18}$$

The integral on the right-hand side of this equation is given in Gradsteyn and Ryzhik[5] as

$$\int \frac{\sin\phi}{\sqrt{1 - k^2 \sin^2\phi}}\, d\phi = -\frac{1}{k}\ln\left[k\cos\phi + \sqrt{1 - k^2\sin^2\phi}\right]_0^{\pi/2}$$

$$= -\frac{1}{2k}\ln\left(\frac{1 - k}{1 + k}\right). \tag{C19}$$

Substituting this into Eq. (C18)

$$\int_0^{\pi/2} \cos\phi\, F(\phi;k)\, d\phi = F\left(\frac{\pi}{2};k\right) + \frac{1}{2k}\ln\left(\frac{1 - k}{1 + k}\right) \tag{C20}$$

and therefore, from Eq. (C15)

$$A_s = \frac{4ab}{2kF\left(\frac{\pi}{2};k\right)} \ln\left(\frac{1 + k}{1 - k}\right). \tag{C21}$$

Keeping in mind that $\beta = b/a$, it is now possible to express Eq.'s (8), (11), (12), and (13) in the body of this report by simple combinations of Eq.'s (C12), (C13), (C21), and (3) as follows:

$$\frac{s}{(S/2)} = \frac{\left(\frac{2}{1 - k^2}\right)\left[E\left(\frac{\pi}{2};k\right) - E(\phi;k)\right] + F(\phi;k) - F\left(\frac{\pi}{2};k\right)}{\left(\frac{2}{1 - k^2}\right)E\left(\frac{\pi}{2};k\right) - F\left(\frac{\pi}{2};k\right)}$$

$$= 1 - \frac{2E(\phi;k) - (1 - k^2)F(\phi;k)}{2E\left(\frac{\pi}{2};k\right) - (1 - k^2)F\left(\frac{\pi}{2};k\right)} . \tag{C22}$$

This is Eq. (5) in the body of this report. From Eq. (C21)

$$\frac{A_s}{4ab} = \frac{1}{2kF\left(\frac{\pi}{2};k\right)} \ln\left(\frac{1 + k}{1 - k}\right) \tag{C23}$$

and from Eq.'s (C23) and (9),

$$\frac{A_s}{4a^2} = \frac{A_s}{4ab}\beta = \frac{A_s}{4ab}\frac{2k}{(1 - k^2)F\left(\frac{\pi}{2};k\right)} . \tag{C24}$$

Therefore,

$$\frac{A_s}{4a^2} = \frac{1}{(1 - k^2)F^2\left(\frac{\pi}{2};k\right)} \ln\left(\frac{1 + k}{1 - k}\right) \tag{C25}$$

Eq.'s (C23) and (C25) are Eq.'s (11) and (12), respectively. Also, from Eq.'s (C13) and (C25),

$$\frac{A_s}{S^2} = \frac{4a^2 \ln\left(\frac{1+k}{1-k}\right)}{(1-k^2)F^2\left(\frac{\pi}{2};k\right)} \quad \frac{\left[(1-k^2)F\left(\frac{\pi}{2};k\right)\right]^2}{a^2\left[4E\left(\frac{\pi}{2};k\right) - 2(1-k^2)F\left(\frac{\pi}{2};k\right)\right]^2}$$

$$= \frac{(1-k^2)\ln\left(\frac{1+k}{1-k}\right)}{\left[2E\left(\frac{\pi}{2};k\right) - (1-k^2)F\left(\frac{\pi}{2};k\right)\right]^2} \tag{C26}$$

This is Eq. (13) in the body of the report.

The tension coefficient C_{T_2} can be calculated from the expression for C_{T_1} as follows.

Solve Eq. (18) for $T_{max}/(\delta\omega^2)$

$$\frac{T_{max}}{\delta\omega^2} = b^2\,\frac{(1+k^2)}{4k^2} = \frac{\beta^2 S^2}{4\left(\frac{S}{2a}\right)^2}\;\frac{(1+k^2)}{4k^2}\,. \tag{C27}$$

Divide through by S^2,

$$\frac{T_{max}}{\delta w^2 S^2} = \frac{1}{4}\,\frac{\beta^2}{\left(\frac{S}{2a}\right)^2}\,\frac{(1+k^2)}{4k^2} \tag{C28}$$

Substituting for β from Eq. (3) and for $S/(2a)$ from Eq. (7) into Eq. (28) yields

$$\frac{T_{max}}{\delta\omega^2 S^2} = \frac{1+k^2}{4\left[2E\left(\frac{\pi}{2};k\right) - (1-k^2)F\left(\frac{\pi}{2};k\right)\right]^2} \tag{C29}$$

Substituting for δ from Eq. (19) gives the expression

$$\frac{\left(\frac{T_{max}}{A_c}\right)}{\rho_e S^2 \omega^2} = \frac{1 + k^2}{4\left[2E\left(\frac{\pi}{2};k\right) - (1 - k^2)F\left(\frac{\pi}{2};k\right)\right]^2} \tag{C30}$$

This is Eq. (23) in the text.

The mass moment of inertia about the axis of rotation is defined by

$$I_z = \delta \int_{-S/2}^{S/2} r^2 ds \tag{C31}$$

which may be written as

$$I_z = \delta b^2 \frac{S}{2} \int_{-1}^{+1} \left(\frac{r}{b}\right)^2 d\left[\frac{s}{\left(\frac{S}{2}\right)}\right]. \tag{C32}$$

and since the integrand is symmetric,

$$I_z = \delta b^2 S \int_{o}^{1} \left(\frac{r}{b}\right)^2 d\left[\frac{s}{\left(\frac{S}{2}\right)}\right]. \tag{C33}$$

Now, on differentiating Eq. (8)

$$\frac{d\left[\frac{s}{\left(\frac{S}{2}\right)}\right]}{d\phi} = - \frac{2\frac{dE(\phi;k)}{d\phi} - (1 - k^2)\frac{dF(\phi;k)}{d\phi}}{2E\left(\frac{\pi}{2};k\right) - (1 - k^2)F\left(\frac{\pi}{2};k\right)} \tag{C34}$$

and hence

$$d\left[\frac{s}{\frac{S}{2}}\right] = -\frac{2\left[1 - k^2 \sin^2 \phi\right]^{1/2} - (1 - k^2)\dfrac{1}{\left[1 - k^2 \sin^2 \phi\right]^{1/2}}}{2E\left(\frac{\pi}{2};k\right) - (1 - k^2)F\left(\frac{\pi}{2};k\right)} d\phi \tag{C35}$$

Substituting for $d\left[\frac{s}{\left(\frac{S}{2}\right)}\right]$ from Eq. (C35) into Eq. (C33) and noting that when $s/\left(\frac{S}{2}\right) = 0$, $\phi = \pi/2$ and when $s/\left(\frac{S}{2}\right) = 1$, $\phi = 0$ and also that

$$\frac{r}{b} = \sin \phi, \tag{C36}$$

the equation for the normalized moment of inertia becomes

$$\frac{I_z}{\delta b^2 S} = -\int_0^{\pi/2} \frac{\sin^2\phi\left(-2\left[1 - k^2 \sin^2 \phi\right]^{1/2} + (1 - k^2)\dfrac{1}{\left[1 - k^2 \sin^2 \phi\right]^{1/2}}\right)}{2E\left(\frac{\pi}{2};k\right) - (1 - k^2)F\left(\frac{\pi}{2};k\right)} d\phi. \tag{C37}$$

From Ref.'s 6 and 4,

$$\int \sin^2 \phi \left[1 - k^2 \sin^2 \phi\right]^{1/2} d\phi = -\frac{1}{3} \sin \phi \cos \phi \left[1 - k^2 \sin^2 \phi\right]^2 + \frac{1 - k^2}{3k^2} F(\phi;k) + \frac{2k^2 - 1}{3k^2} E(\phi;k) \tag{C38}$$

and

$$\int \frac{\sin^2 \phi}{\left[1 - k^2 \sin^2 \phi\right]^{1/2}} d\phi = \frac{1}{k^2}\left[F(\phi;k) - E(\phi;k)\right]. \tag{C39}$$

Substituting from these last two equations into Eq. (C37) and evaluating the results at the limits gives

$$\frac{I_z}{\delta b^2 S} = \frac{1}{3k^2} \frac{(1 - k^2)F\left(\frac{\pi}{2};k\right) - (1 + k^2)E\left(\frac{\pi}{2};k\right)}{(1 - k^2)F\left(\frac{\pi}{2};k\right) - 2E\left(\frac{\pi}{2};k\right)} \tag{C40}$$

Although I_z is normalized by $\delta b^2 S$ in this equation, no consideration has yet been given as to whether this is a useful scaling. One way to look at the problem is to recall the equation for the net torque about the z axis. This is

$$I_z \frac{d\omega}{dt} = \left(\frac{1}{2}\rho_a V^2\right) b A_s C_Q - Q_L \tag{C41}$$

where ρ_a is the air density, C_Q a torque coefficient for the turbine and Q_L is the torque the load exerts on the turbine. Then

$$\frac{d\omega}{dt} = \left(\frac{1}{2}\rho_a V^2\right) C_Q \frac{1}{\left(\frac{I_Z}{bA_s}\right)} - Q_L/I_z = \left[\frac{\rho_a V^2 C_Q}{2\delta}\right] \frac{1}{\left[\frac{I_z}{\delta b A_s}\right]} - Q_L/I_z \tag{C42}$$

Since the first factor on the right has the dimensions of $1/t^2$, it contains the dimensional part of $d\omega/dt$, so that the quantity $I_z/(\delta b A_s)$ can be considered as a non-dimensional scaling factor for the magnitude of $d\omega/dt$. This seems to indicate that this last factor is probably the normalized form of I_z which is of practical interest. Proceeding along these lines, multiply Eq. (C40) through by bS/A_s, and write the result in the form

$$\frac{I_z}{\delta b A_s} = \frac{\beta}{6k^2} \frac{1}{\left(\frac{A_s}{S^2}\right)\left(\frac{S}{2a}\right)} \left[\frac{(1 - k^2)F\left(\frac{\pi}{2};k\right) - (1 + k^2)E\left(\frac{\pi}{2};k\right)}{(1 - k^2)F\left(\frac{\pi}{2};k\right) - 2E\left(\frac{\pi}{2};k\right)}\right]. \tag{C43}$$

Substituting for β from Eq. (3), for A_s/S^2 from Eq. (13) and for $S/(2a)$ from Eq. (7) gives an expression for $I_z/(\delta b A_s)$ which is a function of k only, which for N blades is

$$\frac{I_z}{N\delta b A_s} = -\frac{(1 - k^2)F\left(\frac{\pi}{2};k\right) - (1 + k^2)E\left(\frac{\pi}{2};k\right)}{3k(1 - k^2)\ln\left(\frac{1 + k}{1 - k}\right)} . \tag{C44}$$

This is Eq. (24) in the text.

The r coordinate of the centroid $\bar{r}$ of the area under the troposkien in the first and fourth quadrants can be written symbolically as

$$\bar{r} = \frac{1}{A}\iint r dA \tag{C45}$$

From Fig. 1, the incremental area can be expressed by

$$dA = 2zdr. \tag{C46}$$

Rewriting Eq. (C45) in terms of Eq. (C46),

$$\bar{r} = \frac{4}{A_s}\int_0^b rzdr \tag{C47}$$

which can be normalized to

$$\frac{\bar{r}}{b} = \frac{1}{\left(\frac{A_s}{4ab}\right)}\int_0^1 \left(\frac{r}{b}\right)\left(\frac{z}{a}\right)d\left(\frac{r}{b}\right) \tag{C48}$$

Substituting for z/a from Eq. (1) and sin ϕ for r/b into Eq. (C48) and replacing the limits on $\frac{r}{b}$ with the appropriate limits for ϕ gives

$$\frac{\bar{r}}{b}\left(\frac{A_s}{4ab}\right) = \int_0^{\pi/2} \sin\phi \cos\phi \left[1 - \frac{F(\phi;k)}{F\left(\frac{\pi}{2};k\right)}\right] d\phi$$

$$= \frac{1}{2} - \frac{1}{F\left(\frac{\pi}{2};k\right)} \int_0^{\pi/2} \sin\phi \cos\phi \, F(\phi;k)\, d\phi. \tag{C49}$$

The last integral can be integrated by parts. Let

$$u = -\frac{1}{2}\cos 2\phi; \; du = \sin 2\phi d\phi, \tag{C50}$$

$$v = F(\phi;k); \; dv = \frac{1}{\sqrt{1 - k^2 \sin^2\phi}}\, d\phi. \tag{C51}$$

Then, from

$$\int v du = uv - \int u dv \tag{C52}$$

$$\int_0^{\pi/2} \sin\phi \cos\phi \, F(\phi;k)\, d\phi = \frac{1}{2}\int_0^{\pi/2} \sin 2\phi F(\phi k)\, d\phi$$

$$= \frac{1}{2}\left[-\frac{1}{2}\cos 2\phi F(\phi;k)\right]_0^{\pi/2} + \frac{1}{4}\int_0^{\pi/2} \frac{\cos 2\phi}{\sqrt{1 - k^2 \sin^2\phi}}$$

$$= \frac{1}{4} F\left(\frac{\pi}{2};k\right) + \frac{1}{4}\int_0^{\pi/2} \frac{\cos^2\phi - \sin^2\phi}{\sqrt{1 - k^2 \sin^2\phi}}\, d\phi. \tag{C53}$$

The last integral is given in References 4 and 7, with the aid of which Eq. (C53) can be reduced to

$$\int_0^{\pi/2} \sin\phi \cos\phi \, F(\phi;k)\,d\phi = \frac{1}{4} F\left(\frac{\pi}{2};k\right) + \frac{1}{4}\left\{\frac{1}{k^2} E\left(\frac{\pi}{2};k\right) - \frac{(1 - k^2)}{k^2} F\left(\frac{\pi}{2};k\right)\right.$$

$$\left. - \frac{1}{k^2} F\left(\frac{\pi}{2};k\right) + \frac{1}{k^2} E\left(\frac{\pi}{2};k\right)\right\}$$

$$= \frac{1}{2k^2}\left[E\left(\frac{\pi}{2};k\right) + (k^2 - 1)F\left(\frac{\pi}{2};k\right)\right]. \qquad \text{(C54)}$$

Substituting from this equation into Eq. (C49) and substituting for $A_s/(4ab)$ from Eq. (11) gives

$$\frac{\bar{r}}{b} = \frac{\left(F\,\frac{\pi}{2};k\right) - E\left(\frac{\pi}{2};k\right)}{k\ell n\left(\frac{1 + k}{1 - k}\right)} \qquad \text{(C55)}$$

This is Eq. (25) in the text.

APPENDIX D

CALCULATIONS ASSOCIATED WITH THE APPROXIMATING PARABOLA

The parabolas of concern here are represented by the equation

$$\left(\frac{r}{a}\right) = \beta\left[1 - \left(\frac{z}{a}\right)^2\right]. \tag{D1}$$

The area under the parabola in the first quadrant normalized by a^2 is

$$\frac{A_p}{a^2} = \int_0^1 \left(\frac{r}{a}\right) d\left(\frac{z}{a}\right) = \int_0^1 \beta\left[1 - \left(\frac{z}{a}\right)^2\right] d\left(\frac{z}{a}\right) = \frac{2\beta}{3} . \tag{D2}$$

The swept area is then

$$\frac{A_{sp}}{a^2} = 4\,\frac{A_p}{a^2} = \frac{8\beta}{3} . \tag{D3}$$

The arc length s_p is obtained from

$$\left(\frac{s_p}{a}\right) = \int_0^z \left(1 + \left[\frac{d\left(\frac{r}{a}\right)}{d\left(\frac{z}{a}\right)}\right]^2\right)^{1/2} d\left(\frac{z}{a}\right) = \int_0^1 \left(1 + 4\beta^2\left(\frac{z}{a}\right)^2\right)^{1/2} d\left(\frac{z}{a}\right)$$

$$= \frac{1}{2}\left(\frac{z}{a}\right)\left[1 + 4\beta^2\left(\frac{z}{a}\right)^2\right]^{1/2} + \frac{1}{4\beta}\,\ell n\left\{2\beta\left(\frac{z}{a}\right) + \left[1 + 4\beta^2\left(\frac{z}{a}\right)^2\right]^{1/2}\right\} \tag{D4}$$

The total normalized length S_p/a is obtained by evaluating Eq. (D4) at $\left(\frac{z}{a}\right) = 1$,

$$\frac{S_p}{a} = 2\,\frac{s_p}{a}\bigg]_{z/a=1} = \left[1 + 4\beta^2\right]^{1/2} + \frac{1}{2\beta}\,\ell n\left\{2\beta + \left[1 + 4\beta^2\right]^{1/2}\right\}. \tag{D5}$$

The length and area ratios corresponding to those calculated for the troposkien are easily obtained from Eq.'s (D3), (D4), (D5) and from the definition of β. These are

$$\frac{s_p}{a} = \frac{1}{2}\left(\frac{z}{a}\right)\left[1 + 4\beta^2\left(\frac{z}{a}\right)^2\right]^{1/2} + \frac{1}{4\beta}\,\ell n\left\{2\beta\left(\frac{z}{a}\right) + \left[1 + 4\beta^2\left(\frac{z}{a}\right)^2\right]^{1/2}\right\} \tag{D6}$$

$$\frac{S_p}{2a} = \frac{1}{2}\left[1 + 4\beta^2\right]^{1/2} + \frac{1}{4\beta}\,\ell n\left\{2\beta + \left[1 + 4\beta^2\right]^{1/2}\right\} \tag{D7}$$

$$\frac{s_p}{(S_p/2)} = \left[\frac{2\left(\frac{z}{a}\right)\left[1 + 4\beta^2\left(\frac{z}{a}\right)^2\right]^{1/2} + \frac{1}{\beta}\,\ell n\left\{2\beta\left(\frac{z}{a}\right) + \left[1 + 4\beta^2\left(\frac{z}{a}\right)^2\right]^{1/2}\right\}}{2\left[1 + 4\beta^2\right]^{1/2} + \frac{1}{\beta}\,\ell n\left\{2\beta + \left[1 + 4\beta^2\right]^{1/2}\right\}}\right] \tag{D8}$$

$$\frac{A_{sp}}{4ab} = \frac{A_{sp}}{4a^2\beta} = \frac{8\beta}{3}\,\frac{1}{4\beta} = \frac{2}{3} \tag{D9}$$

$$\frac{A_{sp}}{4a^2} = \frac{2\beta}{3} \tag{D10}$$

$$\frac{A_{sp}}{S_p^{\,2}} = \frac{32\beta^2}{3\left(2\beta\left[1 + 4\beta^2\right]^{1/2} + \ell n\left\{2\beta + \left[1 + 4\beta^2\right]^{1/2}\right\}\right)^2} \tag{D11}$$

The normalized moment of inertia can be derived from

$$I_{zp} = \delta \int_{-\left(\frac{S_p}{2}\right)}^{\left(\frac{S_p}{2}\right)} r^2 ds \tag{D12}$$

$$= 2\delta \int_{o}^{\left(\frac{S_p}{2}\right)} r^2 ds = 2\delta \int_{o}^{a} r^2 \frac{ds}{dz}\, dz \tag{D13}$$

From the differential for the arc length,

$$\frac{ds}{dz} = \left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2} = \left[1 + \left(\frac{d\,\frac{r}{a}}{d\,\frac{z}{a}}\right)^2\right]^{1/2} \tag{D14}$$

Since

$$\frac{r}{a} = \beta\left[1 - \left(\frac{z}{a}\right)^2\right] \tag{D15}$$

$$\frac{d\left(\frac{r}{a}\right)}{d\left(\frac{z}{a}\right)} = -\,2\beta\left(\frac{z}{a}\right). \tag{D16}$$

Substituting from this last equation into Eq. (D14) and then substituting the resulting expression for (ds/dz) and for (r/a) from Eq. (D15) into Eq. (D13) and changing the variable of integration to z/a, gives

$$I_{zp} = 2\delta a^3 \beta^2 \int_{o}^{1} \left[1 - \left(\frac{z}{a}\right)^2\right]^2 \left[1 + 4\beta^2\left(\frac{z}{a}\right)^2\right]^{1/2} d\left(\frac{z}{a}\right) \tag{D17}$$

By expanding the first term in the integrand, the integral can be expressed as the sum of three integrals which are listed in any of several standard tables of integrals. The result of this integration is

$$I_{zp} = \left[1 + 4\beta^2\right]^{1/2}\left[\frac{1}{6} - \frac{5}{96\beta^2} - \frac{1}{256\beta^4}\right] + \left[\frac{1}{4\beta} + \frac{1}{32\beta^3} + \frac{1}{512\beta^5}\right]\ln\left(2\beta + \left[1 + 4\beta^2\right]^{1/2}\right) \tag{D18}$$

In order to compare the results obtained for the moment of inertia of the troposkien with those obtained for the parabola, Eq. (D18) must be normalized by $\delta b A_{sp}$. The result is

$$\frac{I_{zp}}{\delta b A_{sp}} = \frac{\left[1 + 4\beta^2\right]^{1/2}}{8}\left[1 - \frac{5}{16\beta^2} - \frac{3}{128\beta^4}\right] + \frac{3}{8\beta}\left[\frac{1}{2} + \frac{1}{16\beta^2} + \frac{1}{256\beta^4}\right]\ln\left(2\beta + \left[1 + 4\beta^2\right]^{1/2}\right) \tag{D19}$$

where the expression for $A_{sp}/(4a^2)$ has been substituted from Eq. (D10).

The centroid of the area under the parabola in the first and fourth quadrants can be obtained from

$$\bar{r}_p = \frac{4}{A_{sp}}\int_o^b rzdr. \tag{D20}$$

Substituting for z into Eq. (D20) from Eq. (D15) and replacing β by b/a,

$$\bar{r}_p = \frac{4b^2a}{A_{sp}}\int_o^1 \left(\frac{r}{b}\right)\left[1 - \left(\frac{r}{b}\right)\right]^{1/2} d\left(\frac{r}{b}\right)$$

$$= \frac{16}{15}\frac{b^2a}{A_{sp}} \tag{D21}$$

Substituting for A_{sp} from Eq. (D9) into this last equation and dividing the result by b gives the expression for the normalized radius of gyration as

$$\frac{\bar{r}_p}{b} = \frac{2}{5} \quad . \tag{D22}$$

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